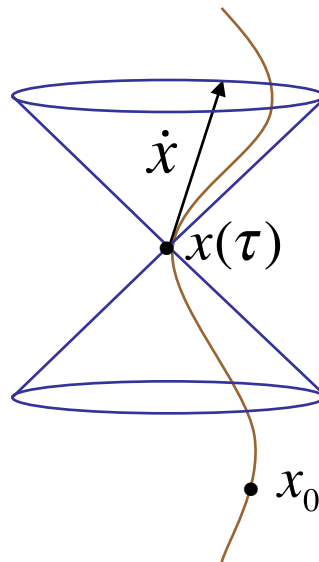


# Hunting for Snarks in Quantum Mechanics

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*“You know, it would be sufficient to really understand the electron!”*

*Einstein*

Santalo 2016

**Snark Hunt:** high stakes search for a *hypothetical physical entity* (**Snark!**)

**Outcomes:**

- **Snark** is **real!** ⇒ scientific **fame** (Nobel Prize)!
- **Snark** is a **boojum!** ⇒ scientific **oblivion** (or scorn)!

Some real snarks & boojums with some of their hunters :

magnetic monopoles (Cabrera, Price)	neutrinos (Pauli)
Higgs boson (LHC)	solar neutrinos (Bachall, Davis)
gravitational waves (Weber, LIGO)	EM waves (Maxwell, Hertz)
tachyons (Sudharshan)	quarks (Gell-Mann)
gravitational worm holes (Wheeler)	blackholes (Hawking)
cold fusion (Fleischmann & Pons)	Bose-Einstein condensation

Current popular snark hunts: **string theory, dark matter**

Pauli **snark test** (sniff, snicker, snide): **“Why it isn’t even wrong!”**

Einstein -- the greatest snark hunter of them all!

On **sniffing out a snark**: “I have a good nose!”

His sniffs of quantum mechanics suggest:

**“Something is rotten in Denmark”**

# The Great Debate on the interpretation of Quantum Mechanics

is centered on meaning of the *wave function*  $\psi$   
and  $\psi^*\psi$  as *probability density* for particle states (**Born Rule**)

## Two major schools:

- The **Copenhagen school** (Bohr, Heisenberg, Pauli, . . . )  
 $\psi$  provides a **complete description** of a physical state.  
*Probability is frequency* expressing an *inherent* randomness in nature.
- The **realist school** (Einstein, de Broglie, Bohm, Jaynes, . . . )  
 $\psi$  provides an **incomplete description** of a physical state  
— only a *statistical ensemble* of similarly prepared states.  
Probability expresses *incomplete knowledge* about the physical state.

[*Bohmian* enclave: <http://www.bohmian-mechanics.net/>]

The **central issue** in the debate was famously articulated by **EPR**

- Does QM admit an experimentally accessible substructure:  
“elements of reality” (Einstein)  
“hidden variables” (Bell)
- Is there a **snark lurking in Quantum Mechanics?**

# The search for hidden structure in Quantum Mechanics

A progress report on the great QM Debate!

- **Pilot Wave Theory** (de Broglie, Bohm): Solutions of Schrödinger's equation determine probable paths for the motion of the electron.
- **Double solution theory**: de Broglie claims that the same equation may have a **singular solution** describing a definite particle path.
- **Electron clock**: a property of the electron proposed by de Broglie to explain wave properties of the electron.
- **Born–Dirac Theory**: Standard theory of the Dirac equation with the Born Rule for a probabilistic interpretation of solutions.
- **Real Dirac Theory**: Reformulation of the Dirac equation in terms of SpaceTime Algebra reveals **hidden geometric structure** relating *spin, complex numbers, electron clock & zitterbewegung!*
- **Maxwell–Dirac Theory**: **Singular solutions** of the Dirac equation for the **electron as a charged hole in the vacuum** with spin and zitter.
- **Fusion of Maxwell–Dirac & Born-Dirac Theories**: **Complementary ontological & epistemological solutions** of the same equation!

Snark!

Snark!

Not Snarks!

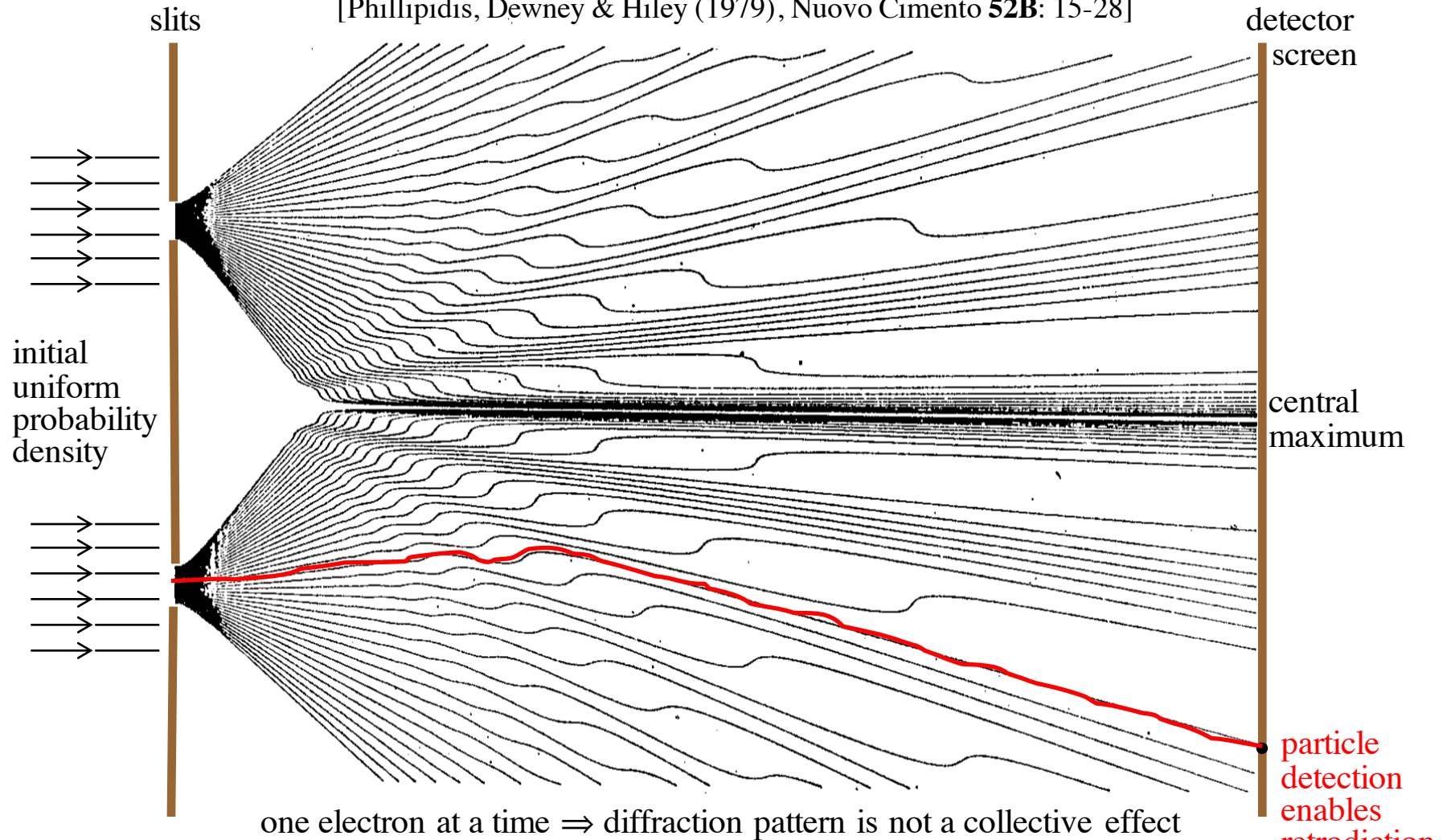
Snark!

Resolution?

# Electron trajectories in the double slit experiment

(from the Schroedinger current)

[Phillipidis, Dewney & Hiley (1979), Nuovo Cimento **52B**: 15-28]



one electron at a time  $\Rightarrow$  diffraction pattern is not a collective effect

$\Rightarrow$  Wave function  $\psi(\mathbf{x}, t)$  carries information about electron state!

particle detection enables retrodiction of path

“relativity is the cornerstone of quantum mechanics.”

Louis de Broglie (according to Georges Lochak )

How reconcile this with standard nonrelativistic QM?!

Two pillars of QM:

- $E = \hbar\omega$       **Planck:** *energy is quantized in frequency!*  
of fields or particle sources?!
- $E = mc^2$       **Einstein:** *mass is energy!*

de Broglie (1924) applied this to the electron:

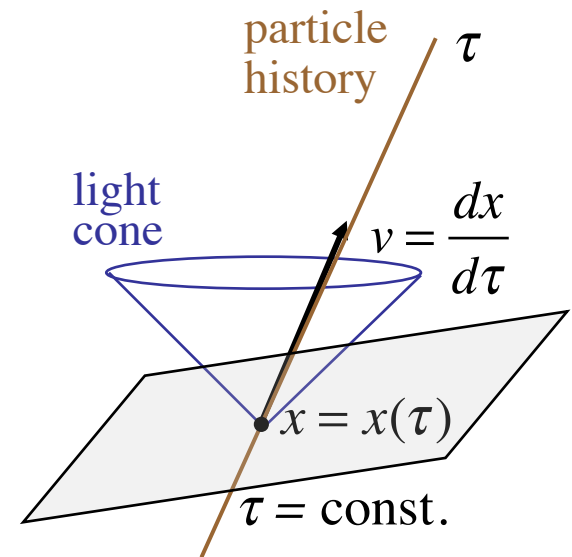
$\Rightarrow$   $\omega_B = \frac{m_e c^2}{\hbar}$       **deBroglie:** *mass is frequency!*

**Voila!**      a particle clock:       $\psi(\tau) = e^{i\omega_B \tau}$

$$\tau = \tau(x) = v \cdot x$$

$$p = m_e c^2 v \quad \Rightarrow \quad \omega_B \tau = \frac{m_e c^2}{\hbar} v \cdot x = \frac{p \cdot x}{\hbar}$$

Congruence of clocks:  $\Rightarrow$   $\psi(x) = e^{\frac{i p \cdot x}{\hbar}} \approx$  Dirac plane wave!!



de Broglie's initial theory *coupled internal periodicity of the electron clock to periodicity of an associated wave.*

**The particle periodicity was discarded almost immediately when Schroedinger introduced his wave equation.**

*Besides how could one read time on a clock with such a high frequency?*

$$\omega_B = \frac{m_e c^2}{\hbar} = 0.77634 \text{ Zs}^{-1}$$

$$\text{Zs}^{-1} = \text{Zetta-Hertz} = 10^{21} \text{ s}^{-1} = (\text{zepto-sec})^{-1}$$

A possibility occurred to **Michel Gouanère** in discussion with **M. Spighel**.

They engaged an experimental team from Lyon to study “Kumakhov radiation” in the 54–110-MeV region on the linear accelerator at Saclay.

Radiation results published in Phys. Rev. B 1988, *but not the clock experiment!*

### Electron Channeling:

Electron beam aligned close to a crystal axis.



Electrons are trapped in orbits spiraling around a single atomic row.

The atomic row can be approximated as a line charge.

The orbits are well described by classical physics,  
especially at high energies.

Scattering is reduced and transmission is greatly increased.

**Michel Gouanère** argued that, if the electron clock is real, a channeled electron should interact resonantly with the crystal periodicity at some energy.

Estimate the resonant energy:

Electron clock in laboratory time  $t$ :  $\psi(\tau) = e^{i\omega_B \tau} = e^{i\omega_L t} = \psi(t)$

$$\Rightarrow \quad \omega_B \tau = \omega_B \frac{t}{\gamma} = \omega_L t \quad \frac{dt}{d\tau} = \gamma = \frac{1}{\sqrt{1 - \mathbf{v}^2 / c^2}}$$

Lab frequency:  $\omega_L = \frac{\omega_B}{\gamma} = \frac{2\pi}{T_L}$       Lab period:  $T_L = \frac{2\pi}{\omega_L}$

Distance traversed during a clock period:

$$d = T_L |\mathbf{v}| = \frac{2\pi h}{m_e c^2} \gamma \frac{m_e |\mathbf{v}|}{m_e} = \frac{hp}{(m_e c)^2}$$

$$\Rightarrow \quad cp = m_e \gamma |\mathbf{v}| = \frac{d(m_e c^2)^2}{hc} = \frac{3.84 \text{ \AA} (0.511034 \text{ MeV})^2}{0.01239852 \text{ MeV-\AA}} = 80.87 \text{ MeV}$$

One day was devoted to the internal clock experiment in 1980,  
but publication was delayed until corrections could be made to the data!



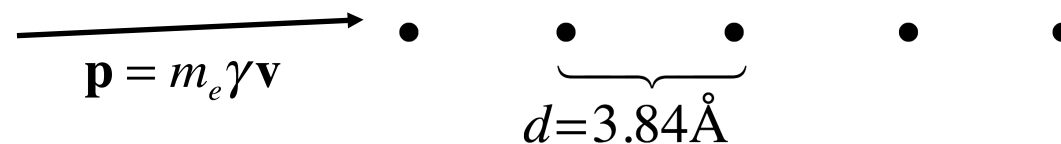
Experimental specifications:

Silicon axis:  $\langle 110 \rangle$ , atomic spacing:  $d = 3.84 \text{ \AA}$

Crystal thickness =  $1 \mu\text{m} = 10^4 \text{ \AA}$  ( $2604d$ )

Search for a transmission resonance in a channeled electron beam  
by scanning a window ( $\pm 1\%$ ) in the momentum range

centered at the predicted resonance momentum  $p = 80.87 \text{ MeV}/c$



Experimental result:

8% transmission dip (in the central peak) found at  $p_{\text{exp}} = 81.1 \text{ MeV}/c$

$$\frac{\Delta p}{p} = \frac{p_{\text{exp}} - p}{p} = \frac{0.23 \text{ MeV}/c}{p} = 0.28\%$$

Estimated calibration error =  $\pm 0.3\%$

Published: *Annales de la Fondation Louis de Broglie*, **30** (1): 109 (2005)

Impact: **NONE!**

To get more visibility, Gouanère submitted to *Physical Review Letters* a version of the experiment supported by a phenomenological calculation used as a guide to design the experiment and interpret its results.

**REJECTED!** as physically implausible! January 2007

But one reviewer suggested a possible mechanism for the effect:

**ZITTERBEWEGUNG!**

$$\omega_Z = \frac{2m_e c^2}{\hbar} = 2\omega_B$$

Problems with interpretation the Dirac equation (1928)

- Mixing of positive & negative energy states
- $\Rightarrow$  oscillations in particle position (Schroedinger, 1930)
- Spin from circulation of electron mass
- Magnetic moment from circulation of electron charge
- Pair creation — Quantum field theory

“The Zitterbewegung Interpretation of Quantum Mechanics”  
Foundations of Physics **20**: 1213-1232 (1990).

Gouanère — Google — Hestenes — met in Paris, May 2007

## What is *Zitterbewegung*?

“The well-known *Zitterbewegung* may be looked upon as a circular motion about the direction of the electron spin with radius equal to the Compton wavelength  $\times 1/2\pi$  of the electron. The intrinsic spin of the electron may be looked upon as the orbital angular momentum of this motion. The current produced by the *Zitterbewegung* is seen to give rise to the intrinsic magnetic moment of the electron.”

*K. Huang, AJP 20: 479 (1952)*

This idea originated with Schroedinger (1930)

- Approved by Dirac soon thereafter!
- Duly recounted in textbooks on relativistic QM today!

However, nothing is made of its **staggering theoretical implications:**

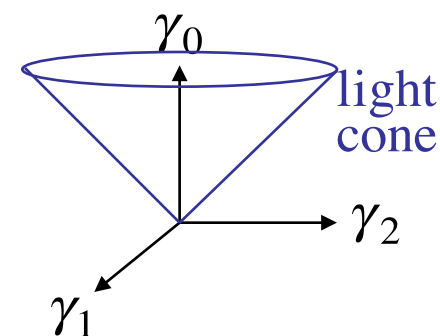
- QM has a **particle substructure** generating magnetic moments!
- Substructure **motion is described by the wave function phase!**
- This motion **must generate a fluctuating electric field!**

**Is this real Snark?? — or just a Boojum!! — How can we tell??**

## SpaceTime Algebra (STA):

Generated by a frame of vectors:  $\{\gamma_\mu\}$

STA  $\xrightarrow[\text{rep}]{\text{matrix}}$  **(Real) Dirac Algebra**



Geometric product:  $\gamma_\mu \gamma_\nu = \gamma_\mu \cdot \gamma_\nu + \gamma_\mu \wedge \gamma_\nu$

Inner product:  $\gamma_\mu \cdot \gamma_\nu = \frac{1}{2}(\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu) = g_{\mu\nu}$  **(metric)**

Outer product:  $\gamma_\mu \wedge \gamma_\nu = \frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$

$$\gamma_0^2 = 1$$

$$\gamma_k^2 = -1$$

STA basis:  $1, \gamma_\mu, \gamma_\mu \wedge \gamma_\nu, i\gamma_\mu, i = \gamma_0 \gamma_1 \gamma_2 \gamma_3$

**scalar, vector, bivector, pseudovector, pseudoscalar**

Vector:  $a = a^\mu \gamma_\mu$

Bivector:  $F = \frac{1}{2} F^{\mu\nu} \gamma_\mu \wedge \gamma_\nu$

Unit pseudoscalar:  $i$

$$i^2 = -1, \quad ia = -ai$$

$$\gamma_0 i = -i \gamma_0 = \gamma_1 \gamma_2 \gamma_3$$

Multivector:  $M = \alpha + a + F + ib + i\beta$

$$\text{dimensions: } 1 + 4 + 6 + 4 + 1 = 16 = 2^4$$

Reverse:  $\tilde{M} = \alpha + a - F - ib + i\beta$

Dual:  $iM = i\alpha + ia + iF - b - \beta$

## Real Quantum Mechanics with STA

geometric imaginary

Real Dirac equation:  $\gamma^\mu (\partial_\mu \psi \gamma_2 \gamma_1 \hbar - q A_\mu \psi) = m_0 \psi \gamma_0$

$$\mathbf{i} \equiv \gamma_2 \gamma_1$$

or:  $\nabla \psi \mathbf{i} \hbar - q A \psi = m_0 \psi \gamma_0$

$$\mathbf{i}^2 = (\gamma_2 \gamma_1)^2 = -1$$

Real wave function:  $\psi = (\rho e^{i\beta})^{\frac{1}{2}} R = \psi(x)$

Rotor:  $R = R(x) \quad R\tilde{R} = 1$

degrees of freedom:  $\uparrow \uparrow \uparrow$   
 $1+1+6=8$

Question: What is the physical meaning of  $\gamma_0$  and  $\mathbf{i} = \gamma_2 \gamma_1$  in Dirac eqn?!

<b>Local observables:</b>	$\psi \gamma_\mu \tilde{\psi} = \rho e_\mu$	comoving frame: $e_\mu = R \gamma_\mu \tilde{R}$
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**Dirac current:**  $\psi \gamma_0 \tilde{\psi} = \rho v$      Answer: velocity  $v = e_0 = R \gamma_0 \tilde{R} = \dot{x}$

Particle conservation:  $\nabla \cdot (\psi \gamma_0 \tilde{\psi}) = \nabla \cdot (\rho v) = 0 \Rightarrow$  congruence of Dirac streamlines:  $x = x(\tau)$

Answer: spin  $s = \frac{\hbar}{2} e_3 = \frac{\hbar}{2} R \gamma_3 \tilde{R}$       $S = \frac{\hbar}{2} R \gamma_2 \gamma_1 \tilde{R} = \frac{\hbar}{2} e_2 e_1 = \frac{\hbar}{2} i e_3 e_0 = i s v$

Question: What is the physical meaning of “hidden variable”  $e_2 = R \gamma_2 \tilde{R}$  ?!

Answer: zitterbewegung!! Along each streamline,  $e_2$  rotates in the  $S$ -plane with the frequency  $d\phi/d\tau = (2m/\hbar \approx 10^{21}$  Hz for a free particle)!

## Anatomy of the Dirac wave function

Dirac:  $\Psi = (\rho e^{i\beta})^{\frac{1}{2}} R$

SpaceTime split:  $R = L U e^{-i\varphi/\hbar}$

Pauli:  $\Psi_P = \rho^{\frac{1}{2}} U e^{-i\varphi/\hbar}$

Schroedinger:  $\Psi_S = \rho^{\frac{1}{2}} e^{-i\varphi/\hbar}$

### Local Observables

$$v = R \gamma_0 \tilde{R} = L \gamma_0 \tilde{L}$$

$$S = \frac{\hbar}{2} R \gamma_2 \gamma_1 \tilde{R} = L i s \tilde{L}$$

$$i s = \frac{1}{2} U i \sigma_3 \hbar \tilde{U} = i \frac{1}{2} \hbar U \sigma_3 \tilde{U}$$

$$i \hbar = i \sigma_3 \hbar = \gamma_2 \gamma_1 \hbar$$

## Lessons learned from Real Dirac Theory

- *Complex numbers are inseparably related to spin in Dirac Theory.*  
 $\Rightarrow$  Spin is essential to interpretation of QM even in Schroedinger Theory.
- *Bilinear observables are geometric consequences of rotational kinematics.*  
 $\Rightarrow$  They are as natural in classical mechanics as in QM.
- *Spin and phase are inseparable kinematic properties of electron motion.*  
 $\Rightarrow$  Wave function **phase is a measure of rotation in the spin plane  $S = i s$ .**

Say that again!!

The **claim** is that the unit imaginary in quantum mechanics

represents a **spacelike bivector**

$$i\hbar = i\sigma_3\hbar = \gamma_2\gamma_1\hbar$$

specifying **fermion spin**

$$is = \frac{1}{2}U i\sigma_3\hbar\tilde{U} = i\frac{1}{2}\hbar U \sigma_3\tilde{U}$$

*This is kind of idea that can ruin a young man's career!*

- *so preposterous* that experts will dismiss it out of hand, usually with a demand for experimental evidence!
- *so compelling* because it is a mathematical **fact** rather than mere speculation!

*So the young man had better be careful when and where he mentions it!*

- **One implication** is that *the Copenhagen interpretation* cannot be correct, because it does not explain how Planck's constant in

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \quad \text{is related to electron spin!}$$

- *Who will believe that?*

# Implications of Real Dirac Theory: the geometry of electron motion with

de Broglie's electron clock in quantum mechanics!

Dirac equation determines a congruence of streamlines,

each a potential **particle history**

$$x = x(\tau)$$

with **particle velocity**

$$\dot{x} = v(\tau) = R\gamma_0\tilde{R}$$

## Spinning frame picture of electron motion

Dirac wave function  $\Psi = (\rho e^{i\beta})^{\frac{1}{2}} R$  determines

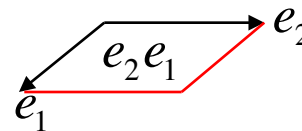
$$\text{Rotor: } R = R(\tau) = R[x(\tau)] = R_0 e^{-\frac{1}{2}\varphi\gamma_2\gamma_1}$$

$$\text{comoving frame: } e_\mu = R\gamma_\mu\tilde{R}$$

phase  $\varphi/2$

$$\text{velocity: } e_0 = R\gamma_0\tilde{R} = v$$

$$\text{Spin: } S = \frac{\hbar}{2} e_2 e_1$$



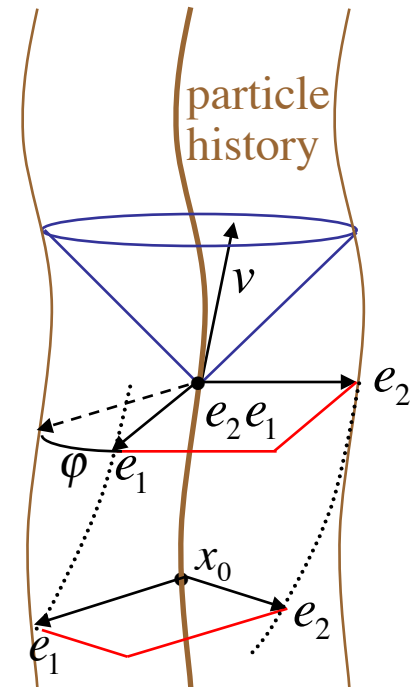
$$e_2 e_1 = R\gamma_2\gamma_1\tilde{R} = R_0\gamma_2\gamma_1\tilde{R}_0$$

Plane wave solution:  $R = R_0 e^{-\frac{1}{2}\varphi\gamma_2\gamma_1} = R_0 e^{-\frac{p \cdot x}{\hbar}\gamma_2\gamma_1}$

$$p = m_e c^2 v \quad \Rightarrow \quad \frac{1}{2}\varphi = \frac{p \cdot x}{\hbar} = \frac{m_e c^2}{\hbar} v \cdot x = \omega_B \tau$$

$$\tau = \tau(x) = v \cdot x$$

$$\omega_B = \frac{m_e c^2}{\hbar} = \frac{1}{2} \frac{d\varphi}{d\tau}$$



Local Observables



Dirac Equation

in terms of

Local Observables

$$\left[ \nabla\psi i\hbar - \frac{e}{c}A\psi = m_e c\psi\gamma_0 \right] \times \tilde{\psi}$$

$$\hbar\nabla\psi\gamma_2\gamma_1\tilde{\psi} - \frac{e}{c}A\psi\tilde{\psi} = m_e c\psi\gamma_0\tilde{\psi}$$

$$\hbar\nabla\psi\gamma_2\gamma_1\tilde{\psi} = \rho P + \nabla(\rho S)$$

$$\nabla(\rho S) = \nabla \cdot (\rho S) + \nabla \wedge (\rho S)$$

Born-Dirac

$$\rho P - \frac{e}{c}A\rho = m_e c\rho v + \nabla \cdot (\rho S)$$

$\rho =$  probability density

Gordon Current

Maxwell-Dirac

$$= \frac{e}{c}A_C + \frac{e}{c}A_M = \frac{e}{c}A_e =$$

Coulomb      Magnetic

Electron vector potential

$\rho^{-1} =$  singular vacuum impedance

$$\psi = (\rho e^{i\beta})^{1/2} R \quad R = L e^{-i\varphi}$$

Density:  $\psi\tilde{\psi} = \rho$  if  $\beta = 0$

Dirac current:  $\rho v = \psi\gamma_0\tilde{\psi}$

Spin density:  $\rho S = \frac{\hbar}{2}\psi\gamma_2\gamma_1\tilde{\psi}$

Momentum density:  $\rho(P = \hbar\nabla\varphi)$

## What is free space?

Maxwell's equation for a homogeneous, isotropic medium

$\varepsilon$  = permittivity (dielectric constant)

$\mu$  = (magnetic) permeability

$$\mathbf{G} = \mathbf{E} + \frac{i}{\sqrt{\mu\varepsilon}} \mathbf{B}$$

$$(\sqrt{\mu\varepsilon} \partial_t - \nabla) \mathbf{G} = 0 \quad \text{Maxwell's equation}$$

$$(\sqrt{\mu\varepsilon} \partial_t + \nabla) \times (\sqrt{\mu\varepsilon} \partial_t - \nabla) \mathbf{G} = 0$$

$$(\mu\varepsilon \partial_t^2 - \nabla^2) \mathbf{G} = 0$$

$$(c^{-2} \partial_t^2 - \nabla^2) \mathbf{G} = 0 \quad \text{Wave Equation}$$

$$c = 1/\sqrt{\mu\varepsilon} = \text{velocity of light in the medium} = \text{free space}$$

D'Alembertian:  $\square^2 = c^{-2} \partial_t^2 - \nabla^2$  Wave operator

Invariant under Lorentz transformations

$\Rightarrow$  *Theory of relativity*

$$\text{But } \sqrt{\frac{\mu}{\varepsilon}} = \rho(x) = ??$$

## Electron as singularity in the physical vacuum

Electromagnetic vacuum defined by:  $\epsilon\mu = \frac{1}{c^2} = \epsilon_0\mu_0$  (Maxwell)

Vacuum impedance undefined:  $Z(x) = \sqrt{\frac{\mu}{\epsilon}} = \frac{1}{\rho(x)} \sqrt{\frac{\mu_0}{\epsilon_0}}$  (E. J. Post)

**Blinder function:**  $\rho = \rho(x) = \sqrt{\frac{\mu}{\epsilon}} \sqrt{\frac{\epsilon_0}{\mu_0}} = e^{-\lambda_e/r}$

Point charge path & velocity:  $z = z(\tau), \quad v = \dot{z} = \frac{1}{c} \frac{dz}{d\tau}$

Retarded distance:  $r = (x - z(\tau)) \cdot v$  with  $(x - z(\tau))^2 = 0$

Classical electron radius  $\lambda_e = \frac{e^2}{m_e c^2}$

**Vector potential:**  
in Maxwell Thry

$$\frac{e}{c} A_e = \frac{e^2}{c\lambda_e} \rho v = \rho m_e c v$$

**Momentum density**  
in Dirac Theory

$$\rho = \psi\tilde{\psi}$$

Unifies Maxwell & Dirac

## *What is an electron?!*

“It is a delusion to think of electrons and fields as two physically different, independent entities. Since neither can exist without the other, there is only *one* reality to be described, which happens to have two different aspects; and the theory ought to recognize this from the outset instead of doing things twice!” – *Einstein*

*Field and particle are all ready unified in the Dirac equation!!*

Dirac equation can be read as an equation for momentum balance:

$$\rho \left( P - \frac{e}{c} A \right) = \rho m_e c v + \nabla \cdot (\rho S)$$
$$= \rho p_e = \frac{e}{c} A_e$$

$$\rho v = \psi \gamma_0 \tilde{\psi}$$

$$\rho S = \frac{\hbar}{2} \psi i \gamma_3 \gamma_0 \tilde{\psi}$$

For "Pilot Wave Solutions":  $\rho = \psi \tilde{\psi} = e^{-\lambda_e / r}$  *Blinder function!*

⇒ Electrons are elementary singularities in the vacuum!

⇒ All elementary particles are topological defects in the vacuum!

## Quantization of Stationary States

$$\text{Split: } z\gamma_0 = ct + \mathbf{r}$$

$$\text{Particle path: } z = z(\tau)$$

$$\text{Period: } z(\tau_n) \cdot \gamma_0 = cT_n$$

$$\text{Periodic orbit: } z(\tau_n) \wedge \gamma_0 = \mathbf{r}(\tau_n) = \mathbf{r}(0)$$

Orbit-clock  
Resonance

$$e_\mu(\tau_n) = R\gamma_\mu\tilde{R} = e_\mu(0) \quad R(\tau_n) = R(0)$$

$$\text{Constant Energy: } E = cP_0 - m_e c^2$$

$$P = (P \cdot \gamma_0 + P \wedge \gamma_0)\gamma_0 = (P_0 + \mathbf{P})\gamma_0$$

Quantization  
Condition

$$\begin{aligned} \oint_0^{\tau_n} P \cdot v d\tau &\equiv \oint P \cdot dz = 0 \\ &= P_0 T_n - \oint \mathbf{P} \cdot d\mathbf{r} = 0 \end{aligned} \quad T_n = \oint_0^{\tau_n} \gamma_0 \cdot v d\tau$$

Magnetic flux  
quantization

$$\oint \mathbf{P} \cdot d\mathbf{r} = 2\pi n\hbar = \oint \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right) \cdot d\mathbf{r} \quad \text{F. London}$$

Electroflux  
quantization

$$P_0 T_n = \oint_0^{\tau_n} \left(p_0 + \frac{e}{c}A_0\right) dt = nh \quad \text{E. J. Post}$$

Resolves Dirac-Sommerfeld puzzle:  $\ell = 0$   $s$ -state

Explains quantum Hall effect

## Classical particles with spin (using STA)

Particle history:  $z = z(\tau)$       Parameter  $\tau$  to be determined

Proper **velocity**:  $u \equiv \dot{z} = \frac{dz}{d\tau}$       constant  $u^2 \geq 0 \Rightarrow \dot{u} \cdot u = 0$

Particle **momentum**:  $p = p(\tau)$       Dynamical **mass**:  $m \equiv p \cdot u$

**Spin** (bivector):  $S = S(\tau)$       (Intrinsic angular momentum)

Essential non-collinearity of velocity and momentum:

$p \neq mu$  because some momentum contained in  $S$ .

Hence relation between  $p$  and  $S$  depends on dynamics.

Kinematical constraint:  $S \cdot u = 0$       (reduces degrees of freedom in  $S$ )

Momentum conservation:  $\dot{p} = f$

Spin conservation:  $\dot{S} = u \wedge p + \Gamma \Rightarrow \dot{S} + m = up + \Gamma$

Orbital angular momentum  $\equiv p \wedge z \Rightarrow$  Total angular momentum:  $J \equiv p \wedge z + S$

$\Rightarrow$  Total angular momentum conservation:  $\dot{J} = f \wedge z + \Gamma$

## Zitter Electron Model

$$z = z(\tau) \quad u = \dot{z} \quad u^2 = \dot{z}^2 = 0$$

$$\dot{u} = -\omega \hat{r} + \frac{q}{m_e} F \cdot u$$

$$\dot{p} = qF \cdot u + \nabla \Phi$$

$$\omega = \frac{2m}{\hbar} = \lambda^{-1} = |r|^{-1}$$

$$r \cdot u = r \cdot p = 0$$

$$\Rightarrow m \equiv p \cdot u = m_e + \Phi$$

$$\Phi = \Phi(\tau, z) = \frac{q}{m_e} S(\tau) \cdot F(z) = q(r_e u) \cdot F$$

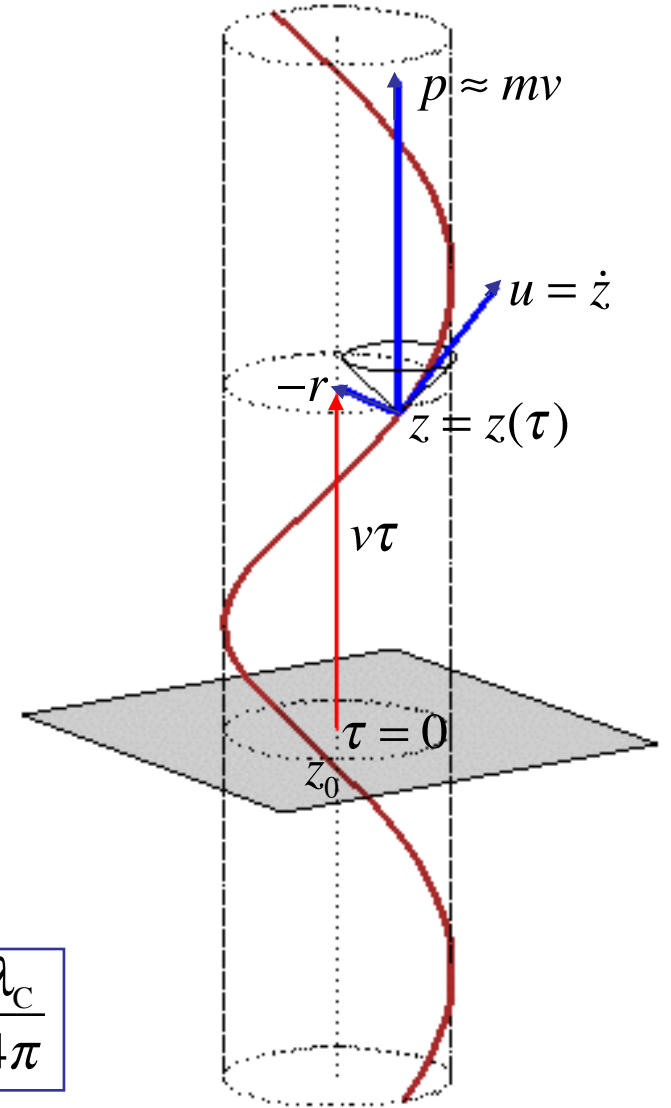
$$S = \frac{\hbar}{2} \hat{r} u = m_e r_e u = r_e \wedge (m_e u) \quad S^2 = 0$$

$$\dot{S} = u \wedge p + \frac{q}{m_e} F \times S$$

$$F \times S \equiv \frac{1}{2} (FS - SF)$$

Scale:

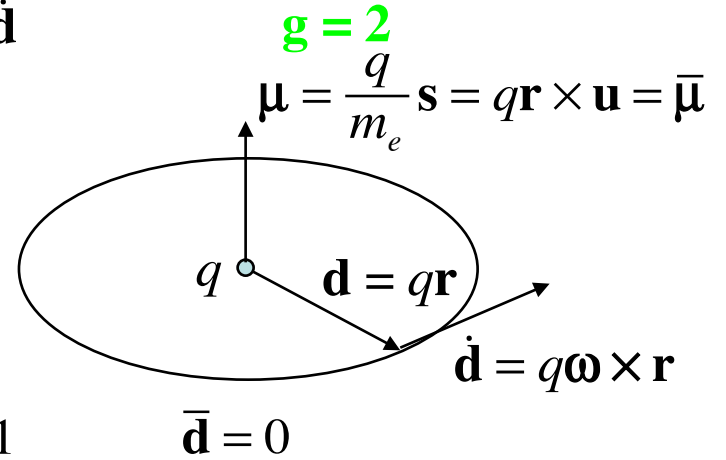
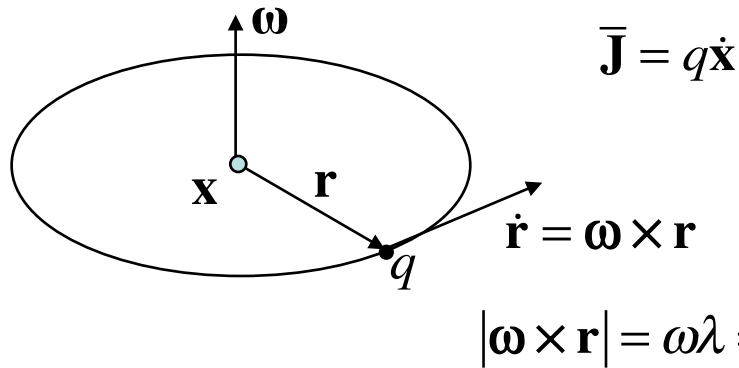
$$\lambda_e = \frac{c}{\omega_e} = \frac{\hbar}{2m_e c} = 1.93079 \times 10^{-3} \text{ \AA} = \frac{\lambda_c}{4\pi}$$



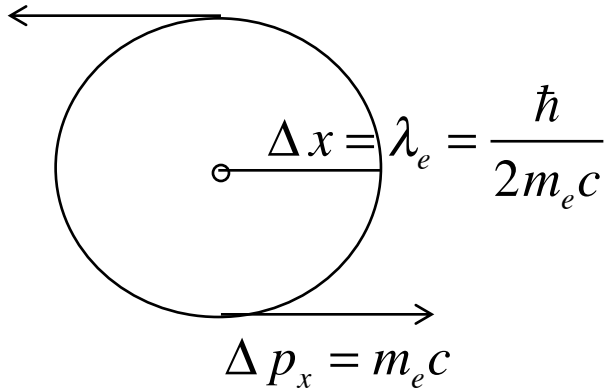
$$\dot{u} = \frac{1}{r} + \frac{q}{m_e} F \cdot u$$

# Zitter in the electron "rest frame"

Charge current:  $\mathbf{J} = q\dot{\mathbf{z}} = q(\dot{\mathbf{x}} + \dot{\mathbf{r}}) = q\dot{\mathbf{x}} + \dot{\mathbf{d}}$

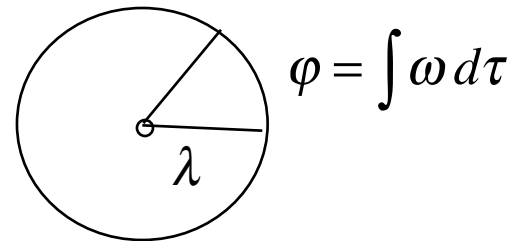


## Spin-Zitter Uncertainty Relations



$$\Delta x \Delta p_x = m_e c \lambda_e = \frac{\hbar}{2}$$

## Zitter frequency & QM phase



$$\omega = \frac{d\varphi}{d\tau} = \omega_e + \frac{2}{\hbar} \Phi = \frac{1}{\lambda}$$

$$m = m_e + \Phi = \frac{1}{2} \hbar \omega$$



**Experimentum crucis: The zitter model explains all results of Gouanère' s electron clock channeling experiment!!**

(Foundations of Physics (2008) **38**: 659-664)

The detailed *explanation*:

- Provides *equations of motion* for channeled electrons with **zitter**
- Explains the clock *interaction mechanism* as resonance with a circulating electric dipole
- Calculates *resonant energies* and explains why  $\omega_Z = \frac{2m_e c^2}{\hbar} = 2\omega_B$   
Parametric resonance!
- Calculates the *width* of the lowest resonance
- Explains the *shift* in resonance energy
- Predicts *spin effects* at higher resolution

The experiment should be refined and repeated to confirm predictions!

**References:**

- “Zitterbewegung in Quantum Mechanics.” Found. Physics (2010) 40:1-54.  
“Reading the Electron Clock.” arXiv:8002.3227v1 [physics.gen-ph]

## CONCLUSION

*Is the electron **really**  
a vacuum singularity  
with clock & zitter?*

Is it a **Snark!**

or a **Boojum?**

BEWARE the consequences!  
Described in the immortal words of  
LEWIS CARROLL:

(1875)

## “The Hunting of the Snark”

as an allegory of scientific research:

The project leader (the *Bellman*)

organizes a scientific team and

defines the research objective: **To discover a Snark!**

The poem personifies the **excitement and perils** of  
scientific **search and discovery**

With the frightening prospect that

*the Snark might turn out to be a **Boojum!***

whereupon the hunter

*“softly and silently vanishes away.”*

**(into scientific oblivion!)**

“The Hunting of the Snark” (edited) — *Lewis Carroll*

“Just the place for a Snark!” the Bellman cried,  
As he landed his crew with care;  
Supporting each man on top of the tide  
By a finger entwined in his hair.

“Just the place for a Snark! I have said it twice:  
That alone should encourage the crew.  
Just the place for a Snark! I have said it thrice:  
What I tell you three times is true.”

. . . .  
Each thought he was thinking of nothing but “Snark”  
And the glorious work of the day;  
And each tried to pretend that he did not remark  
That the other was going that way.

. . . .  
They sought it with thimbles, they sought it with care  
They pursued it with forks and hope;  
They threatened its life with a railway share;  
They charmed it with smiles and soap.

. . . .  
You boil it in sawdust: you salt it with glue:  
You condense it with locusts and tape:  
Still keeping one principal object in view—  
To preserve its symmetrical shape.

. . . .

. . .

(That's exactly the method," the Bellman bold  
In a hasty parenthesis cried,  
"That's exactly the way I have always been told  
That the capture of Snarks should be tried!)

"But oh, beamish friend, beware of the day,  
If your Snark be a Boojum! For then  
You will softly and suddenly vanish away,  
And never be met with again!"

. . .

"There is Thingumbob shouting!" the Bellman said.  
"He is shouting like mad, only hark!  
He is waving his hands, he is wagging his head,  
He has certainly found a Snark!"

. . .

Erect and sublime, for one moment of time,  
In the next, that wild figure they saw  
(As if stung by a spasm) plunge into a chasm,  
While they waited and listened in awe.

. . .

. . . .  
“It’s a Snark!” was the sound that first came to their ears,  
And seemed almost too good to be true.  
Then followed a torrent of laughter and cheers:  
Then the ominous words It’s a Boo——“

Then, silence. Some fancied they heard in the air  
A weary and wandering sigh  
That sounded like”——jum!” but the others declare  
It was only a breeze that went by.

They hunted till darkness came on, but they found  
Not a button, or feather, or mark,  
By which they could tell that they stood on the ground  
Where the Bellman had met with the Snark.

In the midst of the word he was trying to say,  
In the midst of his laughter and glee,  
He had softly and suddenly vanished away——  
For the Snark *was* a Boojum, you see.

The End

*“You know it would be sufficient to really understand the electron!”*  
— Einstein

**GA publications** (more than 100 articles and 10 books)

<<http://modelingnts.la.asu.edu>> <<http://www.mrao.cam.ac.uk>>

***A campaign to Unify the Mathematical Language of Physics***

- “Oersted Medal Lecture 2002: Reforming the mathematical language of physics,” *Am. J. Phys.* **71**: 104-121 (2003).
- “Spacetime physics with geometric algebra,” *Am. J. Phys.* **71**: 691-704 (2003).
- “Gauge Theory Gravity with geometric calculus,” *Foundations of Physics*: June (2005).

***The ground breaking paper on Gauge Theory Gravity:***

- A. Lasenby, C. Doran & S. Gull, “Gravity, gauge theories and geometric algebra,” *Phil. Trans. R. Lond. A* **356**: 487-582 (1998)

***The most comprehensive book on GA:***

Lasenby & Doran, *Geometric Algebra for Physicists*  
(Cambridge: The University Press, 2003).