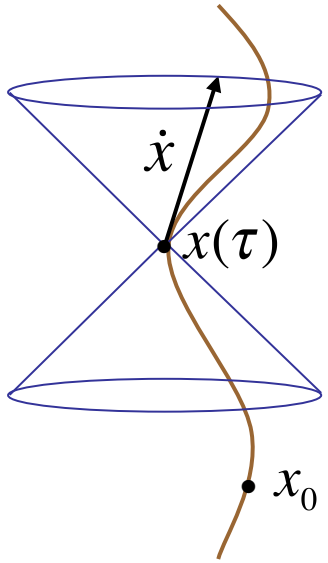
Waves versus Particles:

Deconstructing the electron clock

David Hestenes
Arizona State University



AGACSE 2018

Remembering Waldyr Rodrigues Jr.

First encounter, Gent: **1988**

A colleague's greatest honor! They studied my papers!

Waldr Rodrigues, Jr. Erasmo Recami
Jaimie Vaz Giovanni Salesi

W. Rodrigues, J. Vaz, E. Recami and G. Salesi.
About zitterbewegung and electron structure.
Physics Letters B, 318: 623 - 628, **1993**.

Belated recognition ~ **2005**:

Zitter $\mathbf{i}\psi$ versus $\psi\mathbf{i}$ phase

As leader of the Physics Institute of Campinas State University (UNICAMP)
Waldr brought physics at Campinas to the international stage
by organizing conferences AGACSE 2018
as editor of *Advances in Applied Clifford Algebras*

The Great Debate on the interpretation of Quantum Mechanics

is centered on meaning of the *wave function* ψ
and $\psi^*\psi$ as *probability density for* particle states **(Born Rule)**

Two major schools:

- The **Copenhagen school** (Bohr, Heisenberg, Pauli, . . .)

  ψ provides a **complete description** of a physical state.

  Probability is frequency expressing an *inherent* randomness in nature.

- The **realist school** (Einstein, de Broglie, Bohm, Jaynes, . . .)

  ψ provides an ***incomplete description*** of a physical state

  — only a *statistical ensemble* of similarly prepared states.

  Probability expresses *incomplete knowledge* about the physical state.

  [*Bohmian* enclave: http://www.bohmian-mechanics.net/]

The **central issue** in the debate was famously articulated by *EPR*

- Does QM admit an *experimentally accessible substructure*:

  "elements of reality" (Einstein)

  "hidden variables" (Bell)

- **Are there snarks lurking in Quantum Mechanics?**

Louis de Broglie always insisted that

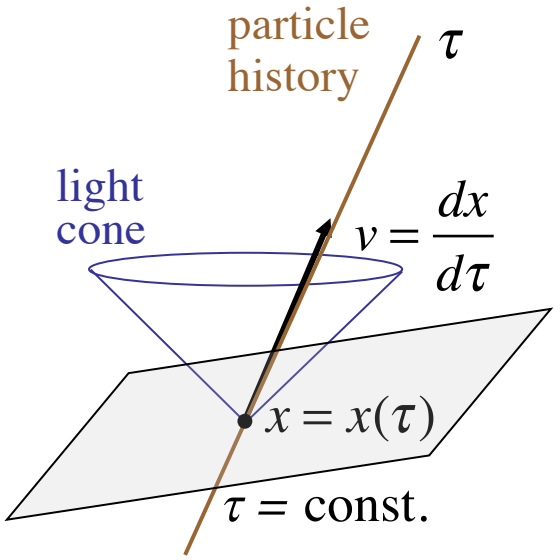
"relativity is the cornerstone of quantum mechanics."

Two relativistic pillars of QM:

- $E = \hbar\omega$ **Planck**: ***energy is quantized in frequency!***
  of fields or particle sources?!
- $E = mc^2$ **Einstein**: ***mass is energy!***

de Broglie (1924) applied this to the electron:

$\Rightarrow$ $\omega_B = \dfrac{m_e c^2}{\hbar}$ **deBroglie**: ***mass is frequency!***



Voila! a **particle clock**: $\psi(\tau) = e^{i\omega_B \tau}$

a **plane wave:** $\tau = \tau(x) = v \cdot x$

$$p = m_e c^2 v \quad \Rightarrow \quad \omega_B \tau = \frac{m_e c^2}{\hbar} v \cdot x = \frac{p \cdot x}{\hbar}$$

Congruence of clocks: $\Rightarrow$ $\psi(x) = e^{\frac{i p \cdot x}{\hbar}}$ $\approx$ a **plane wave** of clocks!!

de Broglie's *clock* was discarded almost immediately
when Schrödinger introduced his wave equation.
de Broglie himself was marginalized by the international physics
community and his clock was all but forgotten,
Except by a band of loyal followers, mostly in France.

Michel Gouanère's **Snark hunt** for **de Broglie's** *electron clock*

Gouanère reasoned that if the clock is real, it must be observable.
But how could one observe time on a clock with such a high frequency?

$$\omega_B = \frac{m_e c^2}{\hbar} = 0.77634\ \mathrm{Zs}^{-1}$$

Zs^{-1}=Zetta-Hertz=$10^{21}\,\mathrm{s}^{-1} = (\text{zepto-sec})^{-1}$

He found it in the ***resonant response*** to crystal periodicity in electron channeling
Gouanère submitted his experimental results to *Physical Review Letters*

REJECTED! as physically implausible! January 2007

"I won't believe the experiment until it is confirmed by Theory!" – *Eddington*

But one reviewer suggested a possible mechanism for the effect:

"The Zitterbewegung Interpretation of Quantum Mechanics"
Foundations of Physics **20**: 1213-1232 (1990).

→ Google
Paris 2007

Brief history of **zitterbewegung**

1923: de Broglie thesis: **e-clock** generates **wave** (Nobel 1929)

1925: Schrödinger reads de Broglie thesis

1926: Schrödinger's wave equation (no clock) (Nobel 1933)

1926: Born rule: $\rho = \psi^*\psi$ = probability density (Nobel 1954)

1928: **Dirac equation**: relativity implies spin?

1929: Negative energy states have opposite charge (proton?}

1930: Schrödinger's **zitterbewegung** frequency: $\omega = 2mc^2/\hbar$
interference between positive and negative energy states

Dirac smells a Snark!

1931: Dirac predicts the **positron!** (Nobel 1933)

1932: Anderson stumbles on the **positron** (Nobel 1936)

Are there more **Snarks lurking** in the **Dirac equation?**

$\Longrightarrow$ My own personal **Snark hunt! Beware!!**

SpaceTime Algebra (STA):

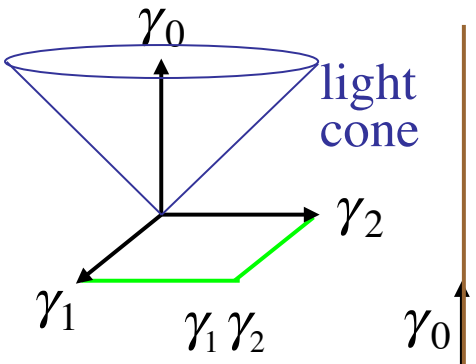
STA $\xrightarrow[\text{rep}]{\text{matrix}}$ **(Real) Dirac Algebra**

Generated by a frame of vectors: $\{\gamma_\mu\}$

Geometric product: $\gamma_0^{\,2} = 1, \qquad \gamma_k^{\,2} = -1 \quad (k = 1,2,3)$

bivector: $\gamma_\mu \gamma_\nu = -\gamma_\nu \gamma_\mu \equiv \gamma_\mu \wedge \gamma_\nu \quad (\mu \neq \nu)$

Orthogonal vectors anticommute!

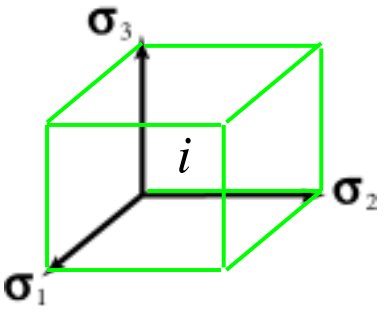


observer history specifies inertial feame

SpaceTime Split: (Real) Pauli Algebra

Spatial vector frame: $\{\boldsymbol{\sigma}_k = \gamma_k \gamma_0\}$

Unit pseudoscalar: $i = \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2 \boldsymbol{\sigma}_3 = \gamma_0 \gamma_1 \gamma_2 \gamma_3$



Many roots of minus one!

pseudoscalar: $i^2 = -1$

bivectors: $(i\gamma_3\gamma_0)^2 = (\gamma_2\gamma_1)^2 = -1$

spacelike vectors: $\gamma_k^2 = -1$

Which root occurs in QM?

Lorentz rotations without matrices or coordinates

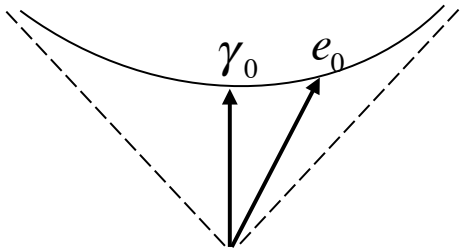
Rotation of a frame: $\gamma_\mu \rightarrow \boxed{e_\mu = R\gamma_\mu \tilde{R}} = a^\eta_\mu \gamma_\eta$

Matrix representation: $a^\eta_\mu = \gamma^\eta \cdot e_\mu = \left\langle \gamma^\eta R \gamma_\mu \tilde{R} \right\rangle$

Rotor R defined by: $R\tilde{R} = 1 \qquad Ri = iR \qquad$ or: $R = e^{\frac{1}{2}B} \qquad \tilde{R} = e^{-\frac{1}{2}B}$

Orthogonality: $e_\mu \cdot e_\nu = \left\langle R\gamma_\mu \tilde{R} R \gamma_\nu \tilde{R} \right\rangle = \left\langle R\gamma_\mu \gamma_\nu \tilde{R} \right\rangle = \gamma_\mu \cdot \gamma_\nu$

SpaceTime Split: $\boxed{R = LU}$



Boost: $e_0 = R\gamma_0 \tilde{R} = L\gamma_0 \tilde{L} = L^2 \gamma_0$

"Pure Lorentz" $L = (e_0 \gamma_0)^{1/2}$

Spatial rotation: $U\gamma_0 \tilde{U} = \gamma_0$

$$\Rightarrow \qquad \mathbf{e}_k \equiv U\boldsymbol{\sigma}_k \tilde{U} = U\gamma_k \gamma_0 \tilde{U} = U\gamma_k \tilde{U} \gamma_0 = \tilde{L} e_k e_0 L$$

Real wave function: $\boxed{\psi = (\rho e^{i\beta})^{\frac{1}{2}} R = \psi(x)}$

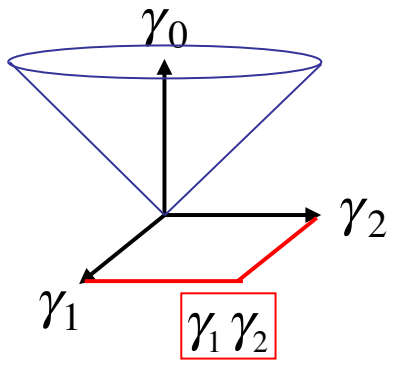
Real Quantum Mechanics with STA

Real Dirac equation: $\gamma^\mu(\partial_\mu\psi\,\gamma_2\gamma_1\hbar - qA_\mu\psi) = m_e\psi\gamma_0$

or: $\nabla\psi\,\mathbf{i}\hbar - qA\psi = m_e\psi\,\gamma_0$

geometric imaginary: $\mathbf{i} \equiv \gamma_2\gamma_1 = i\sigma_3$

$\mathbf{i}^2 = (\gamma_2\gamma_1)^2 = -1$



Real wave function: $\psi = (\rho e^{i\beta})^{\frac{1}{2}} R = \psi(x)$

degrees of freedom: 1+1 + 6 = 8

Rotor: $R = R(x) \quad R\tilde{R} = 1$

Local observables: $\psi\,\gamma_\mu\tilde{\psi} = \rho e_\mu$ comoving frame: $e_\mu = R\gamma_\mu\tilde{R}$

Dirac current: $\psi\,\gamma_0\tilde{\psi} = \rho v$ velocity: $v = e_0 = R\gamma_0\tilde{R} = \dot{x}$

Particle conservation: $\nabla\cdot(\psi\,\gamma_0\tilde{\psi}) = \nabla\cdot(\rho v) = 0 \Rightarrow$ congruence of Dirac streamlines: $x = x(\tau)$

Spin: $s = \frac{\hbar}{2}e_3 = \frac{\hbar}{2}R\gamma_3\tilde{R} \qquad S = isv = \frac{\hbar}{2}ie_3e_0 = \frac{\hbar}{2}e_2e_1 = \frac{\hbar}{2}R\gamma_2\gamma_1\tilde{R}$

Anatomy of the Dirac wave function

Dirac: $\psi = (\rho e^{i\beta})^{\frac{1}{2}} R$

SpaceTime split: $R = LUe^{-\mathbf{i}\varphi/\hbar}$ (boost; spatial rotation)

Pauli: $\psi_P = \rho^{\frac{1}{2}} U e^{-\mathbf{i}\varphi/\hbar}$ (phase)

Schroedinger: $\psi_S = \rho^{\frac{1}{2}} e^{-\mathbf{i}\varphi/\hbar}$

Local Observables

$$v = R\gamma_0\tilde{R} = L\gamma_0\tilde{L}$$

$$S = \frac{\hbar}{2} R\gamma_2\gamma_1\tilde{R} = Li\mathbf{s}\tilde{L}$$

$$i\mathbf{s} = \frac{1}{2} U\, i\boldsymbol{\sigma}_3\hbar\tilde{U} = i\frac{1}{2}\hbar U\,\boldsymbol{\sigma}_3\tilde{U}$$

$$\mathbf{i}\hbar = i\boldsymbol{\sigma}_3\hbar = \gamma_2\gamma_1\hbar$$

Lessons learned from Real Dirac Theory

• *Complex numbers are inseparably related to spin in Dirac Theory*.
⇒ Spin is essential to interpretation of QM even in Schroedinger Theory.

• *Bilinear observables are geometric consequences of rotational kinematics*.
⇒ They are as natural in classical mechanics as in QM.

• *Spin and phase are inseparable kinematic properties of electron motion*.
⇒ Wave function **phase is a measure of rotation in the spin plane S = *i*s.**

Say that again!! Are we looking at a **Snark?!**

The **claim** is that the unit imaginary in quantum mechanics

represents a **spacelike bivector:** $\mathbf{i}\hbar = i\boldsymbol{\sigma}_3\hbar = \gamma_2\gamma_1\hbar$

specifying **fermion spin:** $i\mathbf{s} = \frac{1}{2}U\, i\boldsymbol{\sigma}_3\hbar\tilde{U} = i\frac{1}{2}\hbar U\,\boldsymbol{\sigma}_3\tilde{U}$

This is kind of idea that can ruin a young man's career!

– *so preposterous* that experts will dismiss it out of hand, usually with a demand for experimental evidence!

– *so compelling* because it is a mathematical **fact** rather than mere speculation!

– One implication is that *the Copenhagen interpretation* cannot be correct, because it does not explain how Planck's constant in $\Delta x\,\Delta p_x \geq \frac{\hbar}{2}$ is related to electron spin!

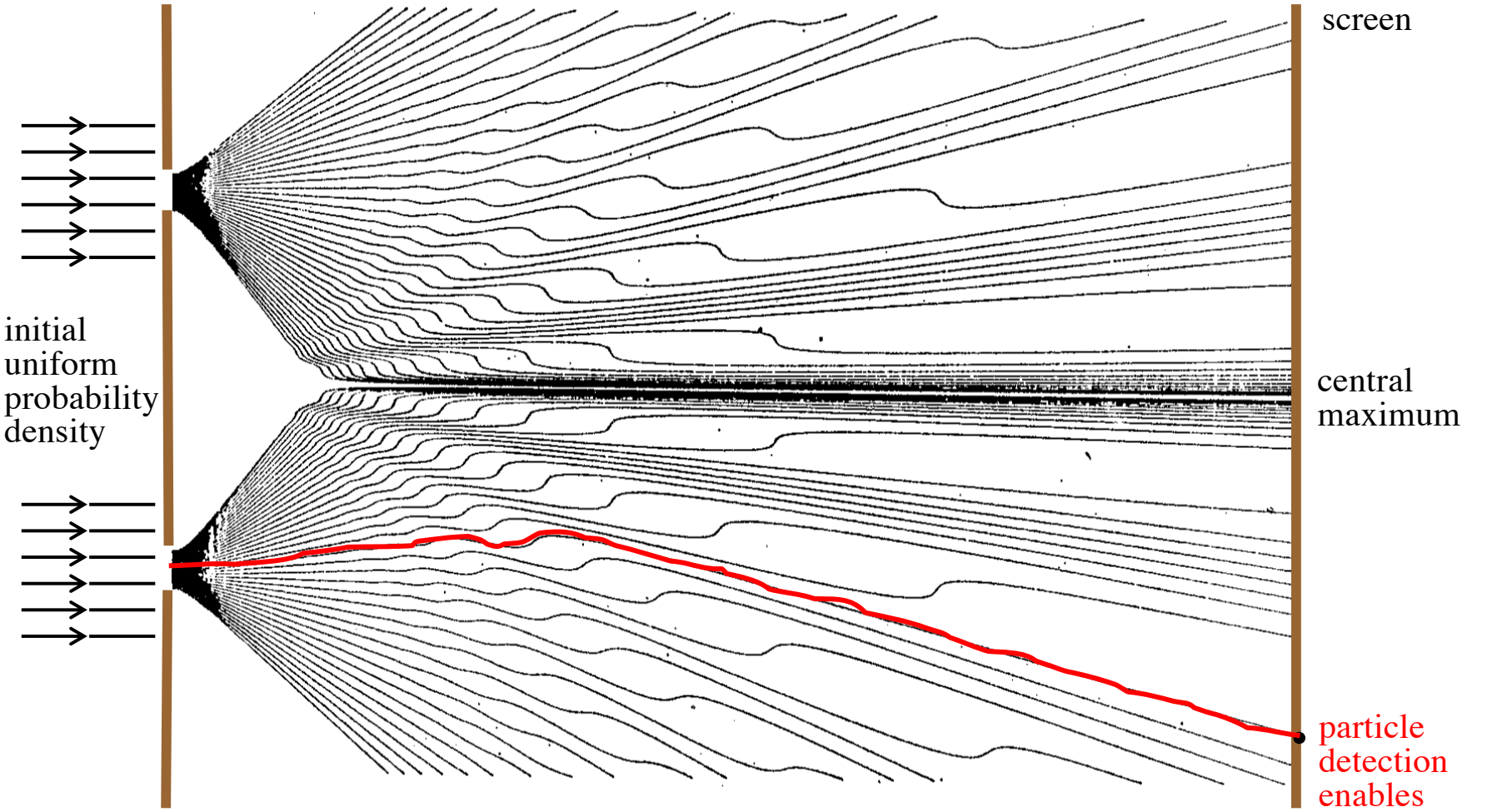
Can the **spin bivector** be generating **zitterbewegung?!**

To find out, we look at **particle paths**.

Pilot Wave $\Rightarrow$ $\psi(\mathbf{x}, t)$ **determines ensemble of electron paths** $\Leftarrow$ **Singular Solutions**

(from the Schrödinger current)

[Phillipidis, Dewney & Hiley (1979), Nuovo Cimento **52B**: 15-28]



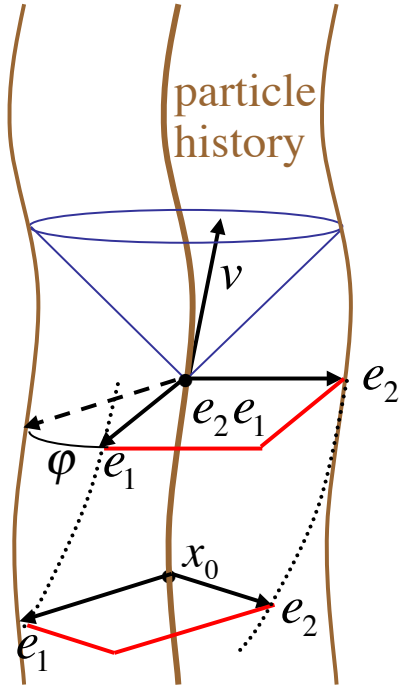
one electron at a time $\Rightarrow$ diffraction pattern is **not a collective effect**

Dirac equation has similar solutions

Implications of Real Dirac Theory: the geometry of electron motion with

de Broglie's **electron clock** in quantum mechanics!

Dirac equation determines a congruence of streamlines,
each a potential particle history $x = x(\tau)$
with particle velocity $\dot{x} = v(\tau) = R\gamma_0\tilde{R}$



Spinning frame picture of electron motion

Dirac wave function $\psi = (\rho e^{i\beta})^{\frac{1}{2}} R$ determines

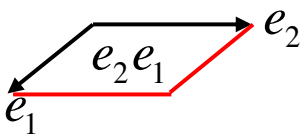
Rotor: $R = R(\tau) = R[x(\tau)] = Ve^{-i\varphi/2}$ phase $\varphi/2$

comoving frame: $e_\mu = R\gamma_\mu\tilde{R}$

Local Observables

velocity: $e_0 = R\gamma_0\tilde{R} = v$

Spin: $S = \frac{\hbar}{2} e_2 e_1$



$e_2 e_1 = R\gamma_2\gamma_1\tilde{R} = R_0\gamma_2\gamma_1\tilde{R}_0$

Plane wave solution: $R = R_0 e^{-\frac{1}{2}\varphi\gamma_2\gamma_1} = R_0 e^{-\frac{p\cdot x}{\hbar}\gamma_2\gamma_1}$

$$\omega_B = \frac{m_e c^2}{\hbar} = \frac{1}{2}\frac{d\varphi}{d\tau}$$

Zitter Solutions of the Dirac equation

The conservation law: $\nabla \bullet (\rho u) = 0$

for the Dirac current: $\psi \gamma_0 \tilde{\psi} = \rho R \gamma_0 \tilde{R} = \rho u$

implies: Dirac *streamlines* ≈ *particle paths*

$z = z(\tau) \qquad u = \dot{z} \qquad u^2 = \dot{z}^2 = 0$

Rotor $R = e^{-I\varphi} V$ with $u = v + e_2$ so $u^2 = 0$

determines a *lightlike helical path* with

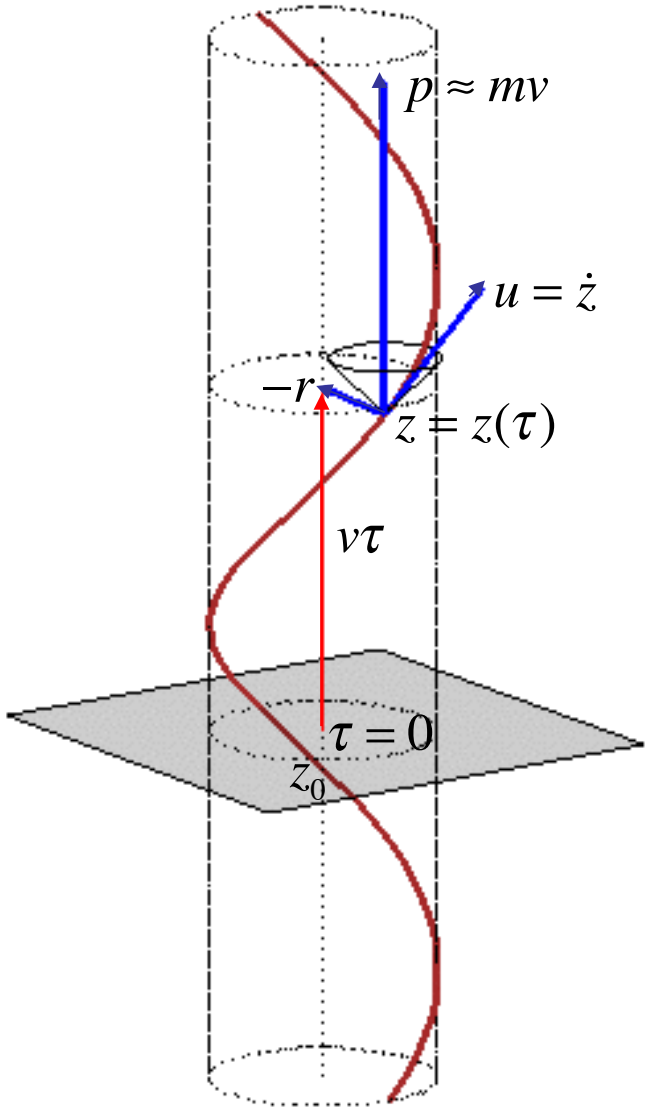
Zitter Radius

$$\lambda_e = \frac{c}{\omega_e} = \frac{\hbar}{2 m_e c} = 1.93079 \times 10^{-3}\ \text{Å} = \frac{\lambda_C}{4\pi}$$

If $I = V \mathbf{i} \tilde{V}$ then $I^2 = \boldsymbol{i}^2 = -1$ and

$$R = e^{-I\varphi} V = V e^{-\mathbf{i}\varphi}$$

reduces to a *rotating frame* solution



Electron as singularity in the physical vacuum

Electromagnetic **vacuum defined** by: $\varepsilon\mu = \dfrac{1}{c^2} = \varepsilon_0\mu_0$ (Maxwell)

Vacuum impedance undefined: $\mu/\varepsilon = \rho(x)$ (E. J. Post)

Vacuum impedance defined by: $\boxed{\rho = \rho(x) = e^{-\lambda_c/r}}$ (Blinder)

Point charge path & velocity: $z = z(\tau), \quad v = \dot{z} = \dfrac{1}{c}\dfrac{dz}{d\tau}$

Retarded distance: $r = (x - z(\tau))\cdot v$ with $(x - z(\tau))^2 = 0$

Classical electron radius: $\lambda_c = e^2/m_e c^2$ fixes scale

Coulomb vector potential A_C: $\boxed{\lambda_c A_C = e\rho v}$ **charge current** (F. London)

charge source: $1/r \xrightarrow[r\to 0]{} \infty \implies \rho = e^{-\lambda_c/r} \xrightarrow[r\to 0]{} 0$ **Vacuum Hole!**

All consistent with classical Maxwell electrodynamics

What flows in solutions of the Dirac equation?

$\rho v =$ ***probability current.*** **Born–Dirac Theory**

$e\rho u =$ ***charge current.*** **Maxwell–Dirac Theory**

Giving the electron a charge requires a particle path

Blinder density: $\rho = e^{-\lambda_c / r} = 0 \xrightarrow{u=\dot{z}(\tau)}$ path

Consistent solution with Blinder ρ **requires** two phases with

lightlike helical path (zitter), and

magnetic moment with $g = 2$

Solution: $R = e^{-I\varphi} V e^{-\mathbf{i}\theta}$

Coulomb Magnetic

$$m_e c \rho u = \frac{e}{c} A_C \qquad \frac{e}{c} A_M = -\nabla \bullet (\rho S)$$

Electron vector potential: $A_e = A_C + A_M$

What is an electron?!

"It is a delusion to think of electrons and fields as two physically different, independent entities. Since neither can exist without the other, there is only *one* reality to be described, which happens to have two different aspects; and the theory ought to recognize this from the outset instead of doing things twice!" – *Einstein*

Field and particle are all ready unified in the Dirac equation!!

Dirac equation can be read as an equation for **momentum balance:**

Dirac equation: $\hbar\nabla\varphi - \frac{e}{c}A = p_c$ $\qquad u = R\gamma_+\tilde{R} = R(\gamma_0+\gamma_2)\tilde{R}$

Gordon current: $\rho p_c = m_e c\rho u - \nabla\bullet(\rho S) = \frac{e}{c}A_e$ $\qquad S = isu = iR\gamma_3\gamma_+\tilde{R}$

"Pilot Particle" solution: $\rho = e^{-\lambda_c/r}$ ***picks out electron path!***

$\Longrightarrow$ Electrons are elementary singularities in the vacuum!

$\Longrightarrow$ All elementary particles are topological defects in the vacuum!

The search for hidden structure in Quantum Mechanics

A status report!

- **Pilot Wave Theory** (de Broglie, Bohm): Solutions of Schrödinger's equation determine probable paths for the motion of the electron.

Snark!

- **Double solution theory**: de Broglie claims that the same equation may have a **singular solution** describing a definite particle path.

Snark!

- **Electron clock**: a property of the electron proposed by de Broglie and used to generate its wave properties.
- **Born–Dirac Theory**: Standard theory of the Dirac equation with the Born Rule for a probabilistic interpretation of solutions.
- **Real Dirac Theory**: Reformulation of the Dirac equation in terms of SpaceTime Algebra reveals **hidden geometric structure** relating ***spin, complex numbers, electron clock & zitterbewegung***!

Not Snarks!

Snark!

- **Maxwell–Dirac Theory**: **Singular solutions** of the Dirac equation for the **electron as a charged hole in the vacuum** with spin and zitter.
- **Fusion of Maxwell–Dirac & Born-Dirac Theories**: **Complementary ontological & epistemological solutions** of the same equation!

Resolution?

Clues for would-be Snarkers (Snark hunters)

- The fabric of spacetime is an electromagnetic vacuum.
- All elementary particles are topological defects in the vacuum.
- The Dirac eq. is a constitutive equation for vacuum singularities.
- Electroweak $SU_2 \otimes U_1$ is a gauge group of the Dirac eq.
- Weak interactions derive from the magnetic spin potential
- Baryons are knotted leptons.
- The Pauli Principle is a consequence of zitter resonances.

References 2018:

D. Hestenes, Quantum Mechanics of the electron particle-clock
Deconstructing the electron clock

The preceding slides were presented at a physics colloquium
U. Oregon, May 2017.
They outline a theoretical perspective developed over a decade,
but I withheld publication because I was unsatisfied.

Miraculously, the **missing piece** I was looking for
appeared suddenly in April 2018
In an article by **Oliver Consa**:
Helical solenoid model of the electron.
Progress in Physics, 14, **2018**.

His model fits perfectly into my theory, moreover,
If confirmed:

Consa's quantitative explanation
for the electron's anomalous moment

will rank among *the great classics of physics*
in simplicity, cogency, and significance!

Worthy of the best by Einstein himself!

”Well, perhaps he was not altogether wrong!”

— *Lord Kelvin*

On hearing of Hertz’s experimental confirmation of Maxwell’s electromagnetic theory of light.

CONCLUSION

Is the electron really
a vacuum singularity
with lightlike zitter?

Is it a **Snark!**

or a **Boojum?**

BEWARE the consequences!
Described in the immortal words of
LEWIS CARROLL: (1875)

"**The Hunting of the Snark**"

as an allegory of scientific research:

The project leader (the *Bellman*)
organizes a scientific team and
defines the research objective: **To discover a Snark!**

The poem personifies the **excitement and perils** of
scientific **search and discovery**

With the frightening prospect that
the Snark might turn out to be a ***Boojum!***
whereupon the hunter
"*softly and silently vanishes away*."
(**into scientific oblivion!**)

"The Hunting of the Snark" (edited) —— *Lewis Carroll*

"Just the place for a Snark!" the Bellman cried,
As he landed his crew with care;
Supporting each man on top of the tide
By a finger entwined in his hair.

"Just the place for a Snark! I have said it twice:
That alone should encourage the crew.
Just the place for a Snark! I have said it thrice:
What I tell you three times is true."

. . .

Each thought he was thinking of nothing but "Snark"
And the glorious work of the day;
And each tried to pretend that he did not remark
That the other was going that way.

. . .

They sought it with thimbles, they sought it with care
They pursued it with forks and hope;
They threatened its life with a railway share;
They charmed it with smiles and soap.

. . .

You boil it in sawdust: you salt it with glue:
You condense it with locusts and tape:
Still keeping one principal object in view—
To preserve its symmetrical shape.

. . .

. . .

(That's exactly the method," the Bellman bold
In a hasty parenthesis cried,
"That's exactly the way I have always been told
That the capture of Snarks should be tried!)

"But oh, beamish friend, beware of the day,
If your Snark be a Boojum! For then
You will softly and suddenly vanish away,
And never be met with again!"

. . .

"There is Thingumbob shouting!" the Bellman said.
"He is shouting like mad, only hark!
He is waving his hands, he is wagging his head,
He has certainly found a Snark!"

. . .

Erect and sublime, for one moment of time,
In the next, that wild figure they saw
(As if stung by a spasm) plunge into a chasm,
While they waited and listened in awe.

. . .

. . .
"It's a Snark!" was the sound that first came to their ears,
And seemed almost too good to be true.
Then followed a torrent of laughter and cheers:
Then the ominous words It's a Boo——"

Then, silence. Some fancied they heard in the air
A weary and wandering sigh
That sounded like"——jum!" but the others declare
It was only a breeze that went by.

They hunted till darkness came on, but they found
Not a button, or feather, or mark,
By which they could tell that they stood on the ground
Where the Bellman had met with the Snark.

In the midst of the word he was trying to say,
In the midst of his laughter and glee,
He had softly and suddenly vanished away——
For the Snark *was* a Boojum, you see.

The End

Is a Beginning

"*You know it would be sufficient to really understand the electron*!"

— Einstein

GA publications (more than 100 articles and 10 books)

<http:\\modelingnts.la.asu.edu> <http://www.mrao.cam.ac.uk>

A campaign to Unify the Mathematical Language of Physics

- "Oersted Medal Lecture 2002: Reforming the mathematical language of physics," *Am. J. Phys.* **71:** 104-121 (2003).
- "Spacetime physics with geometric algebra," *Am. J. Phys.* **71:** 691-704 (2003).
- "Gauge Theory Gravity with geometric calculus," *Foundations of Physics*: June (2005).

The ground breaking paper on Gauge Theory Gravity:

- A. Lasenby, C. Doran & S. Gull, "Gravity, gauge theories and geometric algebra," Phil. Trans. R. Lond. A **356**: 487-582 (1998)

The most comprehensive book on GA:

Lasenby & Doran, *Geometric Algebra for Physicists* (Cambridge: The University Press, 2003).