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ZITTERBEWEGUNG IN RADIATIVE PROCESSES

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ABSTRACT. The zitterbewegung is a local circulatory motion of the electron presumed to be the basis of the electron spin and magnetic moment. A reformulation of the Dirac theory shows that this interpretation can be sustained rigorously, with the complex phase factor in the wave function describing the local frequency and phase of the circulatory motion directly. This reveals the zitterbewegung as a mechanism for storing energy in a single electron, with many implications for radiative processes.

1. INTRODUCTION

Schrödinger was never satisfied with quantum mechanics. He was especially disturbed by the absence of a clear physical mechanism for radiative processes. This issue is of renewed interest today, for powerful new experimental techniques make it possible to investigate radiative processes with unprecedented resolution and precision. Indeed, the discovery of multiphoton ionization and related phenomena has already upset the conventional wisdom about the photoelectric effect [1]. It seems though, that the initial confusion has been cleared up to the satisfaction of most theorists in the field, and extensive theoretical work has produced explanations for most of the new phenomena brought to light in high intensity laser-atom interactions. All this has been accomplished without any fundamental changes in theory, so some regard it as another triumph of quantum electrodynamics. However there are good reasons to be doubtful.

Most explanations in quantum optics are *phenomenological* in the sense that each is based on some *ad hoc* hamiltonian tailored to the problem at hand. A truly *fundamental* explanation, of course, must be derived from the Dirac equation. To be sure, that is not always possible or practical. But it is essential if anything truly new about radiating electrons is to be learned. Phenomenological models for laser-electron interactions are incapable of distinguishing collective activity from the radiative behavior of single electrons. Consequently, I believe, opportunities for discovering fundamentally new knowledge about radiative processes have been missed.

In this note I will argue that a generally overlooked feature of the Dirac theory, the *zitterbewegung (ZBW)*, is the key to understanding radiative processes, and genuinely new physics is to be expected from studying its implications theoretically and experimentally. The argument has three main steps. The first step is a purely mathematical reformulation of thc Dirac theory, so it should be uncontroversial. Nevertheless, the results are so surprising and unfamiliar that most physicists are taken aback. Briefly, the reformulation eliminates superfluous degrees of freedom and reveals a hidden geometric structure in the Dirac theory he imaginary factor $\mathbf{i}\hbar$ in Dirac's equation automatically becomes identified with the electron spin, and the electron wave function has a geometrical interpretation wherein the spin is the angular momentum of a local circulatory molion, that is, the ZBW. This raises serious questions about the interpretation of quantum mechanics which are discussed in the second step of the argument. The final step is concerned with a qualitative discussion of the ZBW in radiative processes and the prospects for new physics.

The key to recognizing the geometric structure of the Dirac theory is reformulating it in terms of *spacetime algebra*, a Clifford algebra providing the optimal encoding of spacetime geometry in algebraic form. Only a "bare bones" account of the spacetime algebra and the ZBW structure of the Dirac theory can be given here. Much more is provided in [2] and the many references therein. Hopefully, the present account can serve as an intelligible introduction to the other articles in these proceedings which employ the spacetime algebra. However, there is no getting around the fact that genuine insight into any mathematical system requires a good deal of time and effort.

2. SPACETIME ALGEBRA.

The spacelime algebra (STA) is generated from spacetime vectors by introducing a suitable rule for multiplying vectors. We begin with the usual Minkowski model of spacetime as a 4-dimensional vector space $\mathcal{M}^4$. In mathematical parlance, STA is the real Clifford algebra of the Minkowski metric on $\mathcal{M}^4$. More specifically, STA is a real associative algebra generated from $\mathcal{M}^4$ by defining an associative product on $\mathcal{M}^4$ with the special property that the square of every vector is scalar-valued. I call this product the *geometric product* to emphasize the fact that it has a definite geometric interpretation which fully characterizes the geometrical properties of spacetime. The geometric product uv of vectors u and v can be interpreted by decomposing it into symmetric and skewsymmetric parts; thus,

$$uv = u\cdot v + u\wedge v\,, \tag{1}$$

where two new products have been introduced and defined by

$$u\cdot v = \tfrac{1}{2}(uv + vu) = v\cdot u\,, \tag{2}$$

$$u\wedge v = \tfrac{1}{2}(uv - vu) = -v\wedge u\,. \tag{3}$$

It follows from the definition of the geometric product that $u\cdot v$ is scalar-valued; indeed, it is the usual inner product defined on Minkowski space. The quantity

$u \wedge v$ is called a *bivector* and it represents a directed plane segment in the same way that a vector represents a directed line segment.

In these proceedings, Ed Jaynes [3] confesses to a long-standing "hang-up" over Eq. (1) which has prevented him from getting into STA. As I have the greatest respect for Ed's intellect and as other physicists may suffer the same hang-up, I shall attempt a cure forthwith. But first, some further discussion will be helpful in preparation.

Note that for orthogonal vectors (as defined by $u \cdot v = 0$), Eq. (1) gives $uv = u \wedge v = -vu$. Thus, the geometric relation of orthogonality is expressed algebraically by an anticommutative geometric product. Similarly, collinearity is expressed by a commutative geometric product. For in that case $u \wedge v = 0$, Eqs. (1) and (2) give $uv = u \cdot v = vu$. In general, (1) shows that the geometric product represents the relative direction of any two vectors by a combination of commutative and anticommutative parts.

To facilitate comparison with the Dirac matrix algebra, it is convenient to characterize the structure of STA in terms of a basis. Let $\{\gamma_\mu;\ \mu = 0, 1, 2, 3\}$ be a righthanded orthononormal basis for $\mathcal{M}^4$ with timelike vector γ_0 in the forward (future) lightcone. In terms of this basis the spacetime metric is expressed by the equations

$$\gamma_0^2 = 1 = -\gamma_k^2 \qquad \text{for} \qquad k = 1, 2, 3\,, \tag{4}$$

and

$$\gamma_\mu \cdot \gamma_\nu = 0 \quad \text{for} \quad \mu \neq \nu\,. \tag{5}$$

Other basis elements of STA, each with a definite geometric interpretaton, can be generated from γ_μ by multiplication. For example, $\gamma_2\gamma_1 = \gamma_2 \wedge \gamma_1$, is a bivector of unit magnitude, as expressed by

$$(\gamma_2\gamma_1)^2 = -1\,. \tag{6}$$

Returning now to Ed's hang-up, he believes that the validity of Eq. (1) requires some new concept of addition. On the contrary, thc concept of additon in Eq. (1) is identical to the one physicists are familiar with in complex numbers. Indeed, Eq. (1) can be read as a separation of a complex number $z = uv$ into real and imaginary parts. To make that obvious, suppose that u and v are spacelike unit vectors subtending an angle θ, so that $u \cdot v = -\cos\theta$, where the minus sign is due to the negative signature. The area of the parallelogram determined by u and v is given by $\sin\theta$ so we can write $u \wedge v = \mathbf{i}\sin\theta$, where $\mathbf{i}$ is the unit bivector for the spacelike plane containing u and v. If γ_1 and γ_2 compose an orthonormal basis for that plane, then $\mathbf{i} = \gamma_2\gamma_1$ and $\mathbf{i}^2 = -1$. Thus, Eq. (1) assumes the familiar form

$$z = uv = -\cos\theta + \mathbf{i}\sin\theta = e^{-\mathbf{i}\theta}\,. \tag{7}$$

Of course, this gives a much richer concept of complex numbers than the ordinary one. The $\mathbf{i}$ has a twofold geometric meaning: It is the generator of rotations in the plane, as can be seen by solving (7) for v; thus,

$$v = -uz = ue^{-\mathbf{i}\theta}\,. \tag{8}$$

It is also the unit directed area element for the plane. Of great importance to us later on will be the fact that all Lorentz rotations are generated by the bivectors of STA. Although STA enriches the concept of complex number, it employs the same old concept of addition.

Formally, addition is defined by the associative and commutative rules. When adding complex numbers, these rules enable us to collect and concatenate real and imaginary parts separately. The same is true when adding combinations of scalars and vectors in STA. I suspect that the underlying cause of Ed's hang-up is the worry that scalars and vectors will get inextricably mixed-up under addition. But addition doesn't mixup real and imaginary parts of complex numbers. Why? Because they are *linearly independent!* That, I believe, is the concept that Ed overlooked in this context. Scalars and vectors don't get mixed-up under addition because they are linearly independent. Indeed, the addition of scalars to vectors is equivalent to augmenting the vectors with an additional component. But if scalars and vectors cannot be concatenated, why add them at all? The answer is the same as for complex numbers: Because multiplication intermixes them, creating valuable new entities such as the spin representations of the rotation group.

There is a certain historical irony that a discussion like this should be necessary in this day and age. The matter was already cleaned up more than a century ago. More than 150 years ago, William Rowan Hamilton worried that a complex number written as the sum of a real and imaginary parts can have no meaning, because unlike things cannot be added. The concept of linear independence had not been invented yet, but it was implicit in his resolution of the problem: He showed that complex algebra is equivalent to a system of operations relating pairs of real numbers. That insight helped gain general acceptance for complex numbers. A decade later, when Hamilton invented the quaternions, the adding of unlike things didn't bother him any more. Our terms *scalar* and *veclor* were coined by Hamilton to denote the two unlike parts of a quaternion (though Hamilton's vectors actually correspond to bivectors in STA). Thus, as originally conceived, scalars and vectors were added. Most physicists became familiar with quaternions from Maxwell's great *Treatise on Electricity and Magnetism* (1873). This included J. Williard Gibbs, who developed the standard vector calculus of today primarily by dismantling quaternions into separate scalar and vector parts. A few generations later the physics community had forgotten about quaternions, and young physicists like Ed were inculcated with the dogmatic proscription against adding scalars and vectors. Trained incapacity! Fortunately, the true relation of vector algebra to quaternions (which Gibbs and everyone else at the time had failed to see) is perfectly clear within the broader perspective of STA. Indeed, as demonstrated in detail elsewhere, both these algebraic systems are fully encompassed and integrated by STA.

Now let us return to discussing the structure of STA. The unit *pseudoscalar* for spacetime is so important that the special symbol i will be reserved to represent it. Its generation by the vector basis is expressed by

$$i = \gamma_0\gamma_1\gamma_2\gamma_3 \,. \tag{9}$$

Geometrically, it represents the unit oriented 4-volume element for spacetime. Its algebraic properties

$$i^2 = -1 \tag{10}$$

$$\gamma_\mu i = -i\gamma_\mu \tag{11}$$

manipulate. Multiplication of (9) by γ_0 yields the pseudovector

$$\gamma_1\gamma_2\gamma_3 = \gamma_0 i \,. \tag{12}$$

Geometrically, this is the (directed) unit 3-volume element for a hyperplane with normal γ_0.

By forming all distinct products of the γ_μ we obtain a complete basis for STA consisting of the $2^4 = 16$ linearly independent elements

$$1, \quad \gamma_\mu, \quad \gamma_\mu \wedge \gamma_\nu, \quad \gamma_\mu i, \quad i \,. \tag{13}$$

It follows that a generic element M in STA, called a *multivector*, can be written in the *expanded form*

$$M = \alpha + a + B + bi + \beta i \,, \tag{14}$$

where α and β are scalars, a and b are vectors and (with summation over repeated indices)

$$B = \tfrac{1}{2}B^{\mu\nu}\gamma_\mu \wedge \gamma_\nu \tag{15}$$

is a bivector with scalar components $B^{\mu\nu}$. Ed should note that (13) implies that the STA is a 16-dimensional linear space, so (14) is equivalent, with respect to addition, to a vector with 16 components. But multiplication is a different story.

The multivector M in (14) can be decomposed into an even part M_+ and an odd part M_-, as expressed by

$$M = M_+ + M_- \,, \tag{16a}$$

$$M_+ = \alpha + B + \beta i \,, \tag{16b}$$

$$M_- = a + bi \,. \tag{16c}$$

A multivector is said to be even (odd) if its odd (even) part vanishes.

For M in the expanded form (14), the operation of reversion in STA is defined by

$$\widetilde{M} = \alpha + a - B - bi + \beta i \,. \tag{17}$$

It follows that for any multivectors M and N,

$$(MN)^{\sim} = \widetilde{N}\widetilde{M}\,. \tag{18}$$

Essentially, reversion amounts to reversing the order of geometric products.

The relation of STA to the Dirac algebra is now easy to state. The Dirac matrices, commonly denoted by the symbols γ_μ can be put into one-to-one correspondence with the basis vectors denoted by the same symbols above. Then the algebra generated by the Dirac matrices *over the reals* is isomorphic to STA. It follows that the geometric meaning attributed to the vectors γ_μ and their products above is inherent in the Dirac algebra, though it is scarcely recognized in the literature. This isomorphism completely defines the geometric content of the Dirac algebra with respect to spacetime. It suggests also that the representation of the γ_μ by matrices is irrelevant to their function in physical theory. This suggestion is confirmed in the next Section by casting the Dirac theory in terms of STA with no reference at all to matrices.

The full Dirac algebra is generated by the γ_μ over a complex instead of a real number field. The fact that the real field suffices to express the full geometric content of the algebra suggests that the 16 additional degrees of freedom introduced by employing a complex field instead are physically irrelevant. This suggestion is also confirmed in the next Section by formulating the Dirac theory without them. Elimination of the irrelevant $\sqrt{-1}$ in the complex number field opens up the possibility of discovering a geometric meaning for the $\sqrt{-1}$ which occurs so prominently in the equations of quantum mechanics. Indeed, equations (4), (6) and (8) show that STA contains *many different roots of minus one*, including three geometrically different types. Each type plays a different role in the Dirac theory.

3. GEOMETRY OF THE DIRAC THEORY.

In the language of STA, the *Dirac equation* can be written in the form

$$\Box\psi\mathbf{i}\hbar - \frac{e}{c}A\psi = m\psi\gamma_0\,, \tag{19}$$

where

$$\Box = \gamma^\mu\partial_\mu\,, \tag{20}$$

$A = A_\mu\gamma^\mu$ is the usual electromagnetic vector potential, and $\mathbf{i}$ is the unit bivector

$$\mathbf{i} = \gamma_2\gamma_1 = i\gamma_3\gamma_0 \tag{21}$$

The Dirac wave function $\psi = \psi(x)$ at each spacetime point $x = x^\mu\gamma_\mu$ is an even multivector with the invariant canonical form

$$\psi = (\rho e^{i\beta})^{\frac{1}{2}}R\,, \tag{22}$$

where i is the unit pseudoscalar, ρ and β are scalars and R satisfies

$$R\widetilde{R} = \widetilde{R}R = 1\,. \tag{23}$$

A brief proof that the above STA representation of the Dirac equation and wave function is mathematically equivalent to the conventional matrix representation is given in the appendix to Gull's article [4].

Equation (19) is Lorentz invariant, despite the explicit appearance of the constants γ_0 and $\mathbf{i} = \gamma_2\gamma_1$ in it. These constants are arbitrarily specified by writing (19). They need not be identified with the vectors of a particular coordinate system, though it is often convenient to do so. The only requirement is that γ_0 be a fixed timelike unit vector, while $\mathbf{i}$ is a spacelike unit bivector which commutes with γ_0. Of course, the γ_0 and $\mathbf{i} = \gamma_2\gamma_1$ in (19) are the same constants that appear in the expressions (25) and (27) below for the Dirac current and the spin.

The most striking thing about (19) is that the role of the unit imaginary in the matrix version of the Dirac equation has been taken over by the unit bivector $\mathbf{i}$, and this reveals that it has a geometric meaning. Indeed, equations (27) and (28) below show that $\mathbf{i}\hbar$ is to be identified with the spin.

Equation (19) may look more complicated than the conventional matrix form of the Dirac equation, but it actually simplifies and enriches the analysis of solutions by making their geometric structure manifest, as is shown in the detailed calculations of Krüger [5]. The key result of the STA formulation is the invariant decomposition (22) of the Dirac wave function. Its geometrical and physical significance is determined by its relation to observables of the Dirac theory, which we specify next.

At each point x, the function $R = R(x)$ in (22) determines a Lorentz rotation (i.e. a proper, orthochronous Lorentz transformation) of a given fixed frame of vectors $\{\gamma_\mu\}$ into a frame $\{e_\mu = e_\mu(x)\}$ given by

$$e_\mu = R\,\gamma_\mu\widetilde{R}\,. \tag{24}$$

In other words, R determines a unique frame field on spacetime. This frame field has a physical interpretation.

First, the vector field

$$\psi\,\gamma_0\widetilde{\psi} = \rho e_0 = \rho v\,, \tag{25}$$

is the *Dirac current*, which according to the Born interpretation, is to be interpreted as a probability current. Accordingly, at each point x, the timelike vector $v = v(x) = e_0(x)$ is interpreted as the probable (proper) velocity of the electron, and $\rho = \rho(x)$ is the relative probability (i.e. proper probability density) that the electron actually is at x.

Second, the vector field

$$\frac{\hbar}{2}\,\psi\,\gamma_3\widetilde{\psi} = \rho\,\frac{\hbar}{2}\,e_3 = \rho s \tag{26}$$

is the *spin* (or polarization) *vector* density. The *spin angular momentum* $S = S(x)$ is actually a bivector quantity, related to the spin vector s by

$$S = isv = \frac{\hbar}{2}\, i\, e_3 e_0 = \frac{\hbar}{2}\, e_2 e_1 = \frac{\hbar}{2}\, R\gamma_2\gamma_1 \widetilde{R}\,. \tag{27}$$

Multiplying this on the right by (22) and using (23), one easily obtains

$$S\psi = \tfrac{1}{2}\psi\gamma_2\gamma_1\hbar\,, \tag{28}$$

which relates the spin S to the bivector $\gamma_2\gamma_1\hbar$.

In general, six parameters are needed to specify an arbitrary Lorentz rotation. Five of the parameters in the Lorentz rotation (24) are needed to specify the direction of the electron velocity v and spin s. This also determines the plane containing e_1 and e_2, as shown in (27). The remaining parameter ϕ determines the orientation of e_1 and e_2 in the e_2e_1 plane. This can be expressed by factoring R into the form

$$R = R_0 e^{\mathbf{i}\phi}\,, \tag{29}$$

where R_0 is determined by the first 5 parameters just mentioned. The parameter ϕ is the *phase* of the wave function, and here we have a geometrical interpretation of the phase. The vectors e_1 and e_2 are not given a physical interpretation in the conventional formulation of the Dirac theory, because the matrix formalism suppresses them completely. But they will be given a kinematical interpretation when the ZBW interpretation is introduced below.

The factorization (22) of the wave function ψ can now be seen as a decomposition into a 6-parameter *kinematical factor* R and a 2-parameter *statistical factor* $(e^{i\beta})^{\frac{1}{2}}$. The parameter ρ is clearly a probability density. The physical interpretation of β raises problems which are yet to be fully resolved. Important insights into this issue are supplied by other articles in these Proceedings. Boudet [6] describes formal properties of β in the geometry of the Dirac theory. Krüger [5] finds new solutions for the hydrogen atom with $\beta = 0$, in sharp contrast to the strange properties of β in the Darwin solution. Gull [4] discusses the *essential* role of β and its relation to spin in matching boundary conditions at a potential step, including the Klein paradox. My own expectation is that a full understanding of β will come only from elaborating the statistical interpretation of the the Dirac theory discussed below. That is why I have relegated β to the statistical factor in the wave function.

The physics (in contrast to the statistics) in the wave function appears to be in the kinematical factor R. Support for this assertion comes from examining the "free particle" solutions of the Dirac equation. There are two distinct types of plane wave solutions with momentum $p = mcv$, an *electron* solution and a *positron* solution. The electron solution has the form

$$\psi = \rho^{\frac{1}{2}} R_0 e^{\mathbf{i}\phi}\,, \tag{30}$$

where ρ and R_0 are constant, but the phase ϕ has the spacetime dependence

$$\hbar\phi = p \cdot x = mcv \cdot x = mc^2\tau \,. \tag{31}$$

Here τ is the *proper time* along *"streamlines"* of the Dirac current, which are straight lines with tangent v orthogonal to the 1-pararneter family of hyperplanes with constant phase constituting a moving plane wave. According to (25) and (26) the electron velocity v and spin s are constant everywhere. But along a streamline ϕ increases uniformly, so the phase factor in (30) rotates e_1 and e_2 in the plane of the spin S with the circular *zitterbewegung frequency*

$$\omega_0 = 2mc^2/\hbar = 1.6 \times 10^{21}\mathrm{s}^{-1} \,. \tag{32}$$

A similar rotation takes place along the streamlines of every solution of the Dirac equation, though, in general, with a variable frequency. Indeed, the decomposition (29) tells us that generally the phase $\phi = \phi(x)$ at each spacetime point x determines a well-defined rotation, not just in some abstract complex plane, but in a definite physical plane, the plane of the spin S at x.

Jaynes [3] tells us that when he looks at the standard Dirac wave function he doesn't see anything that could rotate. This is a striking illustration of how crucially the interpretation of a theory depends on the form of its mathematical representation. The STA formulation makes the rotation inherent in the wave function absolutely explicit. But, as physicists, we are not satisfied with a "mere" mathematical rotation. With Jaynes, we demand to know "What physically is rotating?" Here again the plane wave solution helps gain a vital insight.

The kinematical factor in (29) can be written in the form

$$R = e^{\frac{1}{2}\Omega\tau} R_0 \,, \tag{33}$$

where Ω is the constant bivector

$$\Omega = mc^2 S^{-1} = \frac{2mc^2}{\hbar}\, e_1 e_2 \,, \tag{34}$$

with $e_1e_2 = R\gamma_1\gamma_2\widetilde{R} = -R_0\mathbf{i}\widetilde{R}_0$. Accordingly, Ω is the angular velocity of the frame $\{e_\mu = e_\mu(x(\tau))\}$ as it moves along a streamline. Both $e_0 = v$ and $e_3 = \hat{s}$ are constants of the motion, but

$$\begin{aligned} e_1(\tau) &= e^{\Omega\tau} e_1(0) = e_1(0)\cos\omega_0\tau + e_2(0)\sin\omega_0\tau \\ e_2(\tau) &= e^{\Omega\tau} e_2(0) = e_2(0)\cos\omega_0\tau - e_1(0)\sin\omega_0\tau \end{aligned} \tag{35}$$

where $\omega_0 = |\,\Omega\,|$ is the ZBW frequency. These equations describe the rotation of the frame explicitly.

Now, it has often been suggested on heuristic grounds the electron spin and magnetic moment may be generated by some kind of local circular motion of the electron. This idea cannot be maintained if the electron velocity is identified with the streamline velocity v of the Dirac current, because v is orthogonal to the spin. If the idea is physically correct, the true electron velocity must have a component in the spin plane. Our geometrical representation of the electron plane wave presents us with an obvious choice. We suppose that the velocity of the electron can be identified with the null vector

$$u = e_0 - e_2 \,. \tag{36}$$

Of course, this means that the electron moves with the speed of light, as in Schrödinger's original ZBW model. This hypothesis defines what I call the *zitterbewegung interpretation of the Dirac theory* [2]. It is more general than Schrödinger's idea of the ZBW, for (36) is obviously applicable to any solution of the Dirac equation. In the plane wave case, however, it is easy to integrate.

With the time dependence of e_2 given by (35) and $u = c^{-1}\dot{z}$, Eq. (36) is easily integrated to get the history $z = z(\tau)$ of the electron; thus,

$$z(\tau) = vc\tau + (e^{\Omega\tau} - 1)r_0 + z_0 \tag{37}$$

This is a parametric equation for a lightlike helix $z(\tau) = x(\tau) + r(\tau)$ centered on the streamline $x(\tau) = vc\tau + z_0 - r_0$ with radius vector

$$r(\tau) = e^{\Omega\tau} r_0 = -\frac{c}{\omega_0}\, e_1 = -\frac{c}{\omega_0^2}\,\dot{u} \tag{38}$$

The radius of the helix is half the electron Compton wavelength

$$\lambdabar_0 = c/\omega_0 = \hbar/2mc = 1.9 \times 10^{-13}\mathrm{m}\,. \tag{39}$$

The Dirac current describes the mean velocity over a ZBW period:

$$\bar{u} = e_0 = v\,, \tag{40}$$

so the Compton wavelength is the diameter of ZBW fluctuations about this mean.

From (34) and (38), which imply $\dot{r} = \Omega r$, we find

$$S = mc^2\Omega^{-1} = mr^2\Omega = m\dot{r}r\,. \tag{41}$$

Thus, the spin angular momentum can be regarded as the angular momentum of ZBW fluctuations.

With τ expressed as a function of spacetime position by (31), Eq. (37) describes a spacetime-filling congruence of lightlike helixes centered on Dirac streamlines, with exactly one helix through each spacetime point. In accord with the statistical interpretation of the Dirac wave function, each helix is a possible worldline for the electron, and the modulus of the wave function determines the probability that the electron traverses any particular helix. All these conclusions about the geometry of the plane wave solutions apply generally to every solution of the Dirac equation, though, of course, in the presence of external fields the helixes are bent and distorted. Jaynes [3] has described this view of Dirac solutions aptly as a *"tangle of all the different possible trajectories of a point particle,"* and he exclaimed "I would never in 1,000 years have thought of looking at the Dirac equation in that way!" Never without the STA formulation! Mark again how crucial the mathematical representation is to physical interpretation! The tangled geometry of helixes has been inherent in the Dirac theory all the time; only a suitable representation and definition was necessary to reveal it. Mark that the *ZBW interpretation attributes a purely kinematical meaning to the phase factor*, so the entire factor R in (29) and (22) has a purely kinematical interpretation. This gives the interpretation of the Dirac wave function a maximum degree of coherence.

Berry [7] has given the quantum phase factor a general geometrical interpretation. According to the ZBW interpretation, the phase factor is more literally geometrical than anyone had imagined.

IV. WHAT IS AN ELECTRON, REALLY?

Is the electron a particle always, sometimes, or never? Theorists have come down on every side of this question. A definitive answer is essential to any sort of objectivity attributed to quantum mechanics. I am pleased that Ed Jaynes [3] has come down on the side opposite mine, for the comparison of contrasting interpretations helps highlight the critical issues. I am equally pleased that he has placed Willis Lamb on my side.

The contrasting interpretations that Ed and I defend should not be regarded as dogmatic stances, nor should it overshadow the great extent to which we agree. The participants at this conference know that Ed is well established as one of the world's leading practitioners of quantum mechanics, especially in the domain of quantum optics. All the while, though, he has been one of the most astute critics of quantum mechanics. His criticism has always been based, not on sterile philosophical speculation or mathematical formalism, but on cogent physical reasoning born of his intimate knowledge of both classical and quantum electrodynamics and how they relate to real experimental data. The criticism he presents in these proceedings is only part of the extensive critical evaluation he has presented on other occasions. I find myself in whole-hearted agreement with the entire body of his criticism, and I commend it to any serious student of the foundations of quantum electrodynamics. Ed and I agree that there is great truth in standard quantum mechanics, but the problem is to separate the truth from the fiction. We also agree that the interpretation of the electron wave function is a critical issue. We part company on what to do about it. Though I am sure that Ed agrees that the Dirac theory is somehow more fundamental, like most other theorists in quantum optics, he is content to base his analysis on the Klein-Gordon and Schrödinger approximations to it. I regard that as a grave mistake, for the ZBW structure of the Dirac theory could never be discovered in these approximate theories, even though it is inherent in the phase factor of the wave function, and they thereby inherit a ZBW interpretation from the Dirac theory. With this understood, let us return to the particle issue.

Ed Jaynes, like Asim Barut [8], wants to interpret the electron wave function as describing a real physical entity, rather than just a state of knowledge about the electron as I wist. While I believe that that viewpoint faces insuperable difficulties, I applaud Ed's objective to bring the matter to decisive experimental test, and I agree that this is feasible. I note that the main reason for Ed's stance is that he believes, along with most other physicists, that electron diffraction can *only* be explained as due to "interference of the electron with itself," so with admirable consistency, Ed maintains that the electron must be extended in space like the wave function. This issue of how to explain diffraction is one of the great bugaboos of quantum mechanics, so I will address it from the particle perspective below.

Ed seems to have a hang-up about point particles as well as STA. Let me attempt some therapy. The question "Is the electron a particle?" can and must be addressed at different levels, where different physical issues are at stake. Let me call the first level the *interpretation level*. Here the question is "Does the Dirac theory admit to a coherent particle interpretation which is superior to alternative interpretations?" My answer to this question is, of course, yes! Indeed, I maintain that the ZBW interpretation is lhe only one which comes close to giving a coherent account of all details of the Dirac theory. It is not maintained at this level that the electron *really* is a point particle, but only that the Dirac theory says it is, in the sense that it ascribes to the electron no internal structure and no finite dimensions. The electron spin and magnetic moment are features of electron kinematics rather than internal structure.

In the spirit of Jaynes, it might be suggested that the electron is an extended body and the helixes are world lines of its component parts. This suggestion faces difficulties which seem to rule it out. First, there is an absence of evidence for any interaction among the parts which would be needed to make the body cohere. Second, the dimensions of the body would have to be on the order of a Compton wavelength ($\sim 10^{-13}$m). But this is much too big! Scattering experiments limit the size of the electron (i.e. the size of the domain in which momentum transfer takes place) to less than $\sim 10^{-18}$m [9]. Only the particle interpretation appears to be consistent with this experimental evidence. Additional evidence for the particle interpretation ([2], [10]) is less direct. For example, the explanation for Van der Waals forces requires that atoms are fluctuating dipoles, which they certainly are if electrons are particles orbiting the nucleus rather than laynesian amoebas enveloping the nucleus in static charge clouds. Moreover, the time dilatation in the decay of μ^- particles captured in atomic s-states indicates that they *really are moving with the Bohr velocity* in those states [11]. So must electrons move also.

It seems to me that the Born statistical interpretation is essential for understanding scattering data, and this demands the particle interpretation. How then do we explain the structure in a diffraction pattern? "Interference" is the standard answer! But there is a lesser known alternative which has been propounded vigorously by David Bohm [12] and others for years. This puts Bohm firmly on my side, though Jaynes cites him as a precursor to his amoebic viewpoint. Bohm maintains that the electron is a particle with a definite trajectory and that the wave function determines a family of possible trajectories, just as I do in my ZBW interpretation. On this point, we differ only in details of how the trajectories are determined by the wave function. Bohm uses Schrödinger theory rather than Dirac theory. The trajectories have actually been calculated from the Schrödinger equation for the double slit experiment [13], and the Dirac equation would surely yield essentially the same result. The trajectories flow uniformly through both slits, but thereafter they spread out, bunching up at diffraction maxima and thinning out at minima. When a single electron has been detected on the "diffraction screen," one can (in principle to any desired precision) determine which of the trajectories it *actually* followed and trace the trajectory backwards to determine where the electron passed through one of the slits. In this sense, quantum mechanics allows us to measure definite electron trajectories.

This *description* of electron diffraction is a self-consistent interpretation of the equations of quantum mechanics. It has the great advantage of preserving a consistent particle interpretation, allowing us to maintain that every electron has a continuous (albeit indirectly observable) trajectory. But physicists want more. They want an *explanation* of diffraction, not just a description. They want to identify a causal mechanism underlying diffraction. I don't believe that standard quantum mechanics has achieved that, but I suggest below where the missing mechanism might be found. On the contrary, standard quantum mechanics purports to explain diffraction as a consequence of interference. The possibility of such an explanation is a *mathematical consequence* of the fact that the QM wave equation is linear, so it can be argued in the double slit experiment that the diffraction pattem is caused by interference in the superposition of particular solutions with each slit as source. Accepting this mathematical possibility as physical reality has the strange consequence that the electron must somehow pass through both slits in order to interfere with itself. I maintain that this interpretation buys nothing but trouble, since it is obviously inconsistent with the factually grounded particle interpretation, but it has no greater predictive power. It is as awkward as it is unnecessary. There is actually only one valid solution of the wave equation which matches the boundary conditions in a diffraction experiment, and only that solution is used in the above particle interpretation of the experiment. The subdivision of that solution into interfering particular solutions which separately do not satisfy the boundary conditions can therefore be safely dismissed as a mere mathematical artifice. Accordingly, *the interference explanation of particle diffraction can be dismissed as an artifice introduced in an attempt to manufacture an explanation out of a description.*

Now let us address the particle question at a second, *more fundamental level*. At this level, I agree with Einstein, Rosen and Cooperstock [14] that the electron, as a particle, must not be treated independently of the electromagnetic field but as part of it. The electron in the Dirac theory is an emasculated charged particle, stripped of its own electromagnetic field, like a classical test charge. The central problem of quantum electrodynamics, as recognized by Barut [8] and many others, is to restore the electron's field and deduce the consequences. This is *the self-interaction problem*. Whether, in the ultimate solution to this problem the electron will emerge as a true singularity in the field or some kind of soliton [14] is anybody's guess. One thing is certain, though, the problem is nonlinear. And if quantization is a consequence of this nonlinearity, as I have suggested elsewhere [10], then the self-interaction problem can never be solved with standard quantum mechanics; a more fundamental starting point must be found.

Though the Dirac theory omits the electron's field, it appears to contain vestiges of self-interaction which are valuable clues to a deeper theory. It is widely believed that the electron mass and spin are consequences of self-interaction. But these are properties of the ZBW, so the ZBW itself must derive from self-interaction. Already this suggests [2] that the electron self-field is of magnetic type to produce the spin, and the electron mass comes from a kind of self-inductance of the circular motion.

Interpreted literally, the ZBW motion should be reflected in the electron's electromagnetic field. Specifically, the electron should be the seat of a nonradiating field that oscillates with the ZBW frequency. Call it the *ZBW field*. The usual Coulomb and magnetic dipole fields of the electron are then averages of the ZBW field over a ZBW period. The ZBW frequency is much too high to detect experimentally. However, it has been suggested [10] that many familiar quantum phenomena might be explained as consequences of ZBW resonances. Here are three examples:

(1) *Electron Diffraction*. The ZBW field broadcasts the electron's deBroglie frequency and wavelength to the environment. I submit that in diffraction it is the ZBW field, rather than the electron itself, that feels out the topology of the target and by feedback produces a shift in the phase of the ZBW motion which alters the electron's trajectory. In crystal diffraction, the Bragg angles must then be conditions for resonance between the *broadcasted ZBW wave* and the *feedback wave* scattered off the crystal. They are thus conditions for resonant momentum transfer between the electron and the crystal. An attractive feature of this explanation is that it includes a mechanism for momentum transfer which is missing from conventional explanations of diffraction.

(2) *Atomic States*. An electron bound in an atom is in a ZBW resonant state,

wherein the frequency of the orbital motion is a harmonic of the ZBW frequency. The principal quantum number indexes the harmonics. Now, if the above explanation of diffraction is correct, the electron must be broadcasting a ZBW wave which is scattered resonantly off the nucleus and back to the electron. An atomic state is thus a state of resonant momentum exchange between the electron and itself. This is to say that an electron accelerated by the field of the nucleus is always *radiating continuously, but it is also continuously absorbing its own radiation.* In the ground state, all the radiated energy must be absorbed, since the state is stable. However, in an excited state the *radiation rate* must exceed the *absorption rate* so the state decays. It should therefore be possible to *calculate the lifetime* of the state from the mismatch between these two rates, thus to explain spontaneous emission. All this is just another example of diffraction, with atomic states corresponding to diffraction peaks and "quantum conditions" corresponding to the Bragg law. The main difference between *transient diffraction* by a crystal and *continuous diffraction* within an atom is that the momentum transfer is between two different objects in the first case but between an object and itself in the second.

(3) *Pauli Principle.* Two electrons in the same atomic state will certainly have resonant ZBW frequencies, so momentum exchange via their ZBW fields is to be expected. Thus we have here a natural mechanism for explaining the Pauli principle, Evidently, then, if the electrons have antiparallel spins the ZBW interaction produces a stable two electron state, while if the spins are parallel the state is unstable and so never seen.

Though suggested by the Dirac theory, all this goes well beyond it. It is conceivable that, besides spontaneous emission, the ZBW is responsible for other phenomena, such as the Lamb shift and the anomalous magnetic moment, which are attributed to quantization of the electromagnetic field. Clearly the ZBW idea is pregnant with possibilities for new physics.

V. RADIATIVE PROCESSES.

Ed Jaynes is quite right to assert that if the electron really is a particle but quantum mechanics describes only the behavior of an ensemble, then it must be possible to extract the particle from the ensemble and study it all by itself. The problem with such an extraction, of course, is to ascertain suitable equations of motion for the particle alone because they might differ significantly from equations for the particle behavior within the ensemble. Nevertheless, as a first approach to the problem, I propose to extract a single ZBW worldline from the Dirac theory and interpret it literally as the worldline of an individual electron. Even without equations of motion, we can reason qualitatively about the electron's behavior from what we know about solutions of the Dirac equation. Such reasoning can be quite provocative. Even if it cannot be refined by calculations from exact equations of motion, it may prove useful in guiding the solving and interpreting of the Dirac equatlon.

When an electron is placed in an external field, energy can be absorbed by the ZBW field, producing an increase in ZBW frequency and hence a decrease in the ZBW radius. We know that from solutions of the Dirac equation where binding energies appear in the complex phase factor. Indeed, the *Minimal Coupling Ansatz* can be interpreted as specifying that external fields produce shifts in the ZBW frequency. As Steve Gull puts it, *the electron is a parametric oscillator* with frequency modulated by external fields. A shift in the ZBW frequency ω, is also a shift in electron mass m, because $\hbar\omega = mc^2$ holds generally. The so-called electron *rest mass* is therefore only a lower bound to the electron mass. The electron mass is actually variable and changing all the time in interactions. However, if no external field is present to induce radiation, it may be that the electron can retain a mass greater than its empirical rest mass. In other words, it may be that *energy can be stored in the ZBW of a single free electron*. This possibility can surely be put to experimental test. Indeed, the basic mechanism may have been probed already by recent experiments in quantum optics.

For example, the ZBW mechanism can be deployed to explain *multiphoton ionization* [1]. When a bound atomic electron is irradiated by an intense laser field, the ZBW may absorb a harmonic of the laser frequency, with an attendant increase of electron mass and shrinking of its atomic orbit. Evidently this excited ZBW state is metastable and may persist for some time after the laser field is off. Then the stored energy is liberated either by reradiation or ionization. The phenomenon of *above threshold ionization* [1] shows that the electron (ZBW) may absorb much more than the minimum necessary for ionization. If, indeed, the ZBW is the mechanism for multiphoton and above-threshold ionization, then it must be possible to demonstrate these phenomena in experiments with single atoms. According to the standard explanations, such experiments should not work. It may be added that final state interactions in ionization should be significantly affected by ZBW mass shifts.

This ZBW explanation for the new photoelectric phenomena may appear to be incompatible with conventional explanations. An excellent and accessible explanation grounded in standard quantum electrodynamics is given by André Bandrauk [15]. The idea is that embedding a molecule in a laser alters the effective electronic potential to create a new set of bound states which can be observed with electron probes. This is not necessarily inconsistent with the ZBW explanation, but the putative physical mechanism is quite different. Other experiments will probably be necessary to distinguish between the two possibilities.

Evidence that irradiated single free electrons can absorb harmonics of the laser frequency exists already in the pioneering “stimulated bremsstrahlung” experiments of Tony Weingartshofer [16]. These experiments have been regarded as anomalous in the high intensity laser field, because they cannot be explained by standard arguments. However, I submit that they are just further examples of the ZBW mechanism at work.

To establish unequivocally that energy can be stored in the ZBW of a single free electron, we need cleaner experiments on single electrons. The prediction is that an electron can absorb an nth order harmonic to put it in a metastable state with mass m given by $mc^2 = m_0c^2 + n\hbar\omega_\ell$, where m_0 is the rest mass and ω_ℓ is the laser frequency. Then, under suitable conditions, the electron can be released in this excited state to transport the additional energy until the electron is induced to release it by a collision or some other means. This phenomenon may actually have been observed already in the infamous *Schwartz-Hora effect* described briefly by Jaynes [3]. I hold with Jaynes that this effect is probably real and the possibility deserves to be investigated thoroughly. Our explanations for the effect may appear to be quite different, but remember, I attribute the standard QM phase factor to the ZBW, and the phase factor plays the key role in Jaynes' argument. The main difference is that Jaynes sees the effect as due to coherent action of parts of the electron spread out over a wave packet. The issues are clear. The truth will be found out.

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